Financing time frictions and macroeconomic outcomes

Financing Time Frictions and Macroeconomic Outcomes

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Abstract

This thesis considers several frictions related to the uncertainty firms face when they raise financing to fund future production projects. Following shocks to factors affecting the firm's profitability between the financing stage and the spending stage, the firm's managers may choose to spend less than the amount decided upon at the time of financing (underspending mechanism). Or, if the firm's revenues do not cover the amount due to debtors as a result of the shock, the firm might decide to default on existing debt obligation (endogenous default mechanism). Chapter 2 studies the underspending mechanism in the context of a general equilibrium model. This study is then extended to the case of an economy with multiple industries linked through an input-output network (chapter 3). Chapter 4 provides firm level empirical evidence to the underspending mechanism studied in chapter 2. This is achieved by linking some of the firm's growth indicators - firm-level total factor productivity in particular - to the equity holders' cash rewards.

The theoretical study of the underspending mechanism, links underspending to the level of shocks experienced by the economy and the cost of servicing current debt. Simulation results illustrate the asymmetric effect of underspending and the role it plays in worsening the lows of the business cycle. The aggregation results of chapter 3 prove that the underspending effects remain significant in economies with large number of industries affected by independent shocks, if the provision of intermediary goods is dominated by a small number of industries.

Chapter 5 is dedicated to modelling endogenous defaults in a real business cycle setup. This chapter presents a model able to endogenously generate countercyclical default rates and credit spreads in line with empirical observation, a feature that is lacking in popular financial acceleration general equilibrium models (e.g. Bernanke, Gertler, and Gilchrist (1998) and Carlstrom and Fuerst (1997)).

Dedication

To my parents, with my love and gratitude.

Declaration

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All remaining errors, mistakes and typos are mine.

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Chapter 1

Introduction

The 2008 Great Recession is associated with the bursting of the American housing bubble. The drop in house prices, combined with leverage effects, wiped out a large proportion of the wealth of households in the United States, leading to a decrease in consumer demand. The banking sector was also heavily exposed to the housing market through asset-backed securities. The deterioration in banks' balance sheet that followed the collapse in the value of mortgage-backed securities dented the confidence in the banking sector in a process that culminated with the bankruptcy of Lehman Brothers in September 2008. This led to a rationing of bank credit available to non-financial firms and increased their financing costs. The drop of aggregate output that followed was both rapid and significant. The Federal Reserve and other major central banks reacted swiftly to this chain of events. Short-term interest rates were set to zero, and central banks' balance sheets were used to engage in large-scale purchase programs to replenish and stabilise banks' balance sheets and restore the flow of credit. By spring 2009, the Federal Reserve stress tests concluded that commercial banks had adequate capital levels relative to assets, signalling the end of the financial crisis. The trough of the cycle occurred shortly after. However, the recovery that followed was slow, as confirmed by the European Investment Bank report [European Investment Bank (2016)], "the slowness of the recovery in investment by firms [was] disturbing, particularly given the extraordinary monetary stimulus". This is the story of a particularly asymmetric cycle. The downturn was violent and brief, and the recovery long and slow. Gertler and Gilchrist (2018) provide a more detailed timeline of the Great Recession as well as a survey of the main prevailing theories explaining it. These surveyed theories share the common feature that borrowers' net worth affects their access to credit and thus their ability to spend.

In chapter 2 of this dissertation, rather than following the literature and focusing on the borrowers net worth, I study a mechanism based on the way firms consider their financing and spending problems when credit is not constrained. Within the studied framework, firms decide their level of financing first. Following unexpected drops in productivity between the financing stage and the time of spending, the firm may choose to spend less than the previously set financing. This does not happen when productivity matches or beats expectations. The studied mechanism asymmetrically affects the business cycle: it worsens the slumps of the cycle and does not operate following shocks that improve productivity. The reason for considering situations where credit is not rationed is two-fold. First, it has been documented that larger corporates with direct access to credit markets took advantage of this access and tapped the bond market to counteract the decline in bank lending in the early stages of the Great Recession. As a result, these firms could maintain stable overall debt levels throughout the credit cycle (Adrian, Colla, and Shin (2012)). In addition, as mentioned above, liquidity was readily available after the early stages of the Great Recession with no significant recovery in terms of output growth. The studied mechanism can contribute to understanding the collapse in output by firms that maintained access to credit markets in the early stages of the Great Recession whilst providing a potential explanation for the slow recovery that followed the financial crisis.

I assume that the firms may decide to spend less than the financing raised if their productivity is affected by large unpredicted shocks in the elapsed time between the financing and spending stages. At the financing stage, the firm considers current financing costs and expectations of future productivity and production costs to set the financing level. The financing level is set such that the marginal impact of financing on expected profit matches the cost of financing as proxied by interest rates. After executing its financing operations and gaining more knowledge about its own productivity and the production costs it faces, the firm reviews its expenditure problem and is usually constrained by its financing level. How much the firm is going to spend is decided by weighing two options: either spending all the cash available on production or returning a portion of it to investors. Returning cash to investors implies no gain and no loss, or equivalently a return on investment equal to one. On the other hand, the return on spending targeted at the financing stage was supposed to be large enough to cover the cost of financing - the targeted return is higher than one. As long as no unexpected adverse shocks hit the firm's productivity, the marginal profit to shareholders from spending on production would remain positive at the financing constraint, and the profit maximising firm would spend all the cash available to finance production. However, if productivity unexpectedly drops low enough between the financing stage and the time of spending, the firm might find it more advantageous not to use all the available financing. After a large enough unexpected drop in productivity, returning some cash to investors becomes more profitable than investing all the funding available in the production process. The studied firm based financial mechanism only functions following unexpected deteriorations in firms' profits and thus provides a potential explanation for the asymmetry of the business cycle.

To help explain the significant drop in output during the Great Recession, the model presented in chapter 2 requires a sizeable unexpected drop in productivity. Fernald (2014) has documented an important decline in the United States' total factor productivity (TFP) growth between 2005 and 2008. More recent productivity data published by the Bureau of Labour Statistics indicate a decline in utilisation-adjusted TFP growth after the recession. Other factors contributing to an unexpected deterioration in firms' profits can achieve a similar effect and push firms to spend less than the previously raised financing. For example, a surprise collapse of consumer demand following a deterioration in the households' net worth as in Mian and Sufi (2012) can play a similar role to unexpected drops in productivity.

Chapter 3 builds on the work of Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and extends the model of chapter 2 to a multiple industry setup where firms are linked through input-output relationships. The chapter discusses the impact of the nature of the production network on the main mechanism of the model presented in chapter 2 and provides aggregation results for two notable productions networks. I show that the business cycle asymmetries generated by the underspending mechanism do not necessarily fade away as the number of industries making the economy grows large. The nature of the production network connecting the industries plays a crucial role in the aggregation of the effects of underspending. For instance, one can prove that the probability of having all industries entering an unconstrained spending mode remains stable as disaggregation increases when the industries are connected through a "star" network. A star network describes an input-output structure where a single industry provides all others with all their intermediary input needs. Conversely, when the provision of the intermediate goods is equally supplied by all industries (symmetric fully connected network), underspending becomes very unlikely when the number of industries grows large.

Chapter 4 provides a firm-level empirical validation to the underspending mechanism studied in chapters 2 and 3. This mechanism suggests that firms tend to distribute the excess cash to shareholders when they become less productive or face less attractive investment opportunities. To present some empirical validation of this claim, chapter 4 builds on the work of Fama and French (2001), Grullon and Michaely (2004) and others to study the relationship between the firm's productivity and the level of cash it diverts towards shareholders in the form of dividends and share buybacks. The corporate finance literature includes many empirical studies stipulating that firms tend to distribute less cash to shareholders when they invest in their production process through research and development, capital expenditure or hirings. The empirical work in chapter 4 brings further confirmation to this while focusing on the impact of firm-level total factor productivity. I find that firms with low productivity are more likely to distribute cash to shareholders and that they tend to distribute more cash relative to their size, as measured by market capitalisation.

Chapter 5 considers another possible reaction by the firm to an unexpected drop in productivity, namely failing or refusing to pay previously contracted debt obligation, i.e. bankrupting on existing debt contracts. The chapter presents a general equilibrium model with endogenous defaults where default rates and credit spreads are countercyclical without the need to introduce an exogenous process driving credit risk. Endogenous defaults happen when the revenues generated by the borrowing firms do not cover their debt obligations. Countercyclical fluctuations of default rates and credit spreads throughout the business cycles are achieved by assuming an intermediate/final production structure. The countercyclical behaviour of the intermediate input prices implies countercyclical revenues of intermediate producers, thus pushing their bankruptcy rates higher in the lows of the business cycle.

The general equilibrium model presented in chapter 5 assumes the existence of a class of intermediate production firms that employ no labour and that provide a final good producing representative firm with intermediate input. While the representative final good producer firm relies on equity investments for financing, the intermediate producers finance their operations using bank loans. Defaults are therefore limited to the intermediate production sector that employs no labour. This simplifies the model by disentangling the bankruptcy problem from the labour demand problem. Intermediate producers have heterogeneous productivities with two components: an idiosyncratic component that determines whether any particular firm ends up defaulting or not and a systemic component that is linked to aggregate TFP and that contributes to the fluctuations of the size of the defaulting subset of firms. To enable the calibration of the model to reproduce historical volatilities of credit spreads, I introduce costly adjustments to the size of intermediate production. The costly adjustment to the production size implies that resetting the loan demand by the borrowing firms also comes at a cost. This adjustment cost affects the demand side of the loan market and is used to generate reasonable credit spread dynamics.

The remainder of this introductory chapter provides a brief and selective summary of the main strands of literature related to this work and highlights this thesis' contributions to the existing literature.

Contributions and Related Literature

This work is related to many strands of economic literature. First, there is the empirical literature studying the recent slump in private investment. The model I study in chapter

2 assumes that capital is accumulated by households outside the firm while focusing on the firms' production spending. Nevertheless, capital expenditure is generally a good indicator of firms' behaviour towards their total expenditure, as firms tend to cut capital spending early when entering a low spending period. Through the empirical study of the relationship between firm-level productivity and shareholders rewards (dividends and share buybacks), this work is also related to the corporate finance literature concerned with explaining the relationships between dividends and share buybacks on the one hand and corporate investment activities on the other. In addition, the main mechanism presented in chapter 2 and extended in chapter 3 only functions during the slumps of the business cycle, so it is related to the empirical literature concerned with business cycle asymmetries. Then there is the "financial acceleration" literature that highlights the relationship between finance and the real economy and shows the "accelerating" effect of finance-related mechanisms on the business cycle. This literature is related to the content of chapters 2, 3 and 5. Furthermore, this work studies the effect of the firm's financial behaviour as industry disaggregation increases. It is therefore related to the literature concerned with aggregation of idiosyncratic productivity shocks. Finally, through the content of chapter 5 of this thesis, this work relates to the effort of modelling credit risk, in the form of endogenous firm bankruptcies, in the context of modern general equilibrium models.

The sluggish recovery that followed the financial crisis has been linked to weak corporate investment. IMF (2015) shows that private business is responsible for most of the slump in global investment and argues that this is caused by weak demand and, in some countries, by financial frictions and political uncertainty. European Investment Bank (2016) explains that real investment in Europe fell sharply between 2008 and 2013 before starting a recovery that led to it being back to pre-crisis levels in core European countries by 2016 while it remained substantially lower than pre-2008 levels in Europe's peripheral countries. In their study of private investment in the American economy, Gutierrez and Philippon (2016) show that investment in the United States is low relative to measures of profitability and valuation (particularly Tobin's Q). On the other hand, the authors argue in another paper that investment has been low in Europe in the post-crisis period, but in line with the relatively low levels of Tobin's

Q.[1] The assumptions underlying the underspending mechanism of chapter 2 imply that productivity affects firms' spending asymmetrically throughout the business cycle: productivity does impact the firms' demand for financing throughout the cycle, but the effect on spending is only immediate following unexpected and large negative shocks. In other words, I assume that financial investment is consistent with Q-theory while real investment is determined by financial investment unless there are large unforeseen drops in productivity. I also show that the underspending mechanism is more likely to operate when nominal interest rates are low. The possibility of underspending later provides the firm with protection against large drops in profits, thus increasing the firm's financing demand. The option to underspend affects the firm's financing demand more when underspending is more likely, as is the case when interest rates are low.

Another related empirical topic is the asymmetry of the business cycle, a fact that has been argued by economists as early as the work of Burns and Mitchell (1946). The seminal work of Neftci (1984) provided a statistical test that proved that "the behaviour of unemployment is characterised by sudden jumps and slow drops", using a finite state Markov process framework. Other authors continued to focus on this topic. For instance, Ramsey and Rothman (1996) link business cycle asymmetry to the notion of "time irreversibility". The latter concept is intuitively related to the mechanism I study in chapter 2 where predetermined financing levels constrain firm expenditure only when productivity is higher than a minimum level that is mainly determined by past interest rates.

This work builds on the existing finance literature concerned with explaining the levels of cash distributed by firms towards shareholders. These cash distributions take two important forms: dividends and share buybacks. Jensen (1986) argues that, when the firm is facing less attractive investment opportunities, a conflict of interest arises between shareholders and managers, with the latter having an incentive to keep more resources under their control and thus not distributing free cash flows. Share repurchases in this context can work as a tool to reassure markets about this potential conflict of interests. Grullon and Michaely (2004) find

[1] Dottling, Gutierrez Gallardo, and Philippon (2017).

that share repurchasing firms reduce their current levels of capital expenditures and research and development expenses and that their cash balances significantly decline. This corroborates the deterioration of the investment opportunities hypothesis. They also find that, contrary to what is suggested by the signalling hypothesis, the markets do not always react positively to the announcement of share repurchases, as market participants are not always aware of the reduction of investments opportunities available to the firm before the share buyback programme is announced. Hribar, Jenkins, and Johnson (2006) show that there is a strong discontinuity in the probability of accretive share repurchases around the consensus earnings per share (EPS) expected by financial analysts. Firms that would have narrowly missed the analysts' consensus EPS are much more likely to increase their share repurchase activity to positively affect their EPS and meet the consensus than those who narrowly beat the consensus EPS. Almeida, Fos, and Kronlund (2016) exploit this discontinuity to show that EPS-motivated share buybacks are associated with reductions in employment and investments. Fama and French (2001) focus on a more usual way chosen by firms to divert cash towards shareholders: dividends. They study the decline in the distribution of dividends by publicly traded firms in the last 20 years of the twentieth century and relate the said decline to several contributing factors, including a change in the characteristics of public firms (firms go public earlier in their development process) and the emergence of competing ways to pay shareholders (e.g. share buybacks). The authors also document an empirical inverse relationship between the firms' propensity to pay dividends and the investment opportunity it faces. Since the early 80s, share repurchases make a significant part of the cash flows directed by firms towards investors. In chapter 4, I construct an index combining both dividends and cash repurchases to account for all cash flows directed towards equity investors as opposed to those being invested in the production processes. This follows the literature concerned with total cash flow distributed by firms to equity investors. Bagwell and Shoven (1989) give an early account of the increasing roles of share redistribution and take-overs as ways to distribute cash from firms towards equity investors and suggest that yields of return on equity investments should account for these ways of cash distribution. Robertson and Wright (2006) use a total cash flow index that takes into account dividends, share repurchases and net share issues and use the constructed index to predict stock returns. Imrohoroglu and Tuzel (2014) use total factor productivity (TFP) to predict equity returns and show that while TFP underperforms other indicators such as the market to book ratio in predicting equity returns, low productivity firms earn a significant premium over high productivity firms in the following year. In chapter 4, I use various firm indicators to explain the propensity of firms to divert cash towards shareholders. Following existing literature, these indicators include investments in capital, research and development and employment. In this regard, my results provide further validation of the idea that firms react to lower investment opportunities by diverting cash towards shareholders. In order to provide an empirical foundation to the mechanism presented in chapter 2, I show that when used in conjunction with a set of other investment indicators, firm-level total factor productivity helps explain the levels of cash diverted to equity investors. To this effect, I present evidence from repeated cross-sectional logit regressions documenting the propensity of firms to pay shareholders. Additionally, I present dynamic panel data regressions explaining the payout size when the firm decides to pay.

The term ”financial acceleration” was first coined in a 1996 paper by Bernanke, Gertler and Gilchrist, in which the authors focused on the agency costs of lending and their endogenous changes over the business cycle (Bernanke, Gertler, and Gilchrist (1996)). The seminal paper of Kiyotaki and Moore (1997) linked the firms' ability to borrow to the value of the collateral they own and thus to the fluctuation of the business cycle, providing a new motivation for the financial acceleration mechanism. A later paper by Bernanke, Gertler, and Gilchrist (1998) builds on Kiyotaki and Moore's idea and develops a New Keynesian model where the cost of external funding of firms depends on their net worth. In chapters 2 and 3, I focus on the firm's decision process regarding its own financing and spending when credit is abundant. Separating the financing problem from the spending problem gives the firm a more realistic economic agency as it controls the use of the funds raised in the financing stage. The firm exerts its control on real investment and can choose to invest less under low productivity conditions even if credit is readily available. This can deepen and lengthen economic downturns and, unlike many other financial accelerators studied in the literature, influences the business cycle in an asymmetric fashion. Unlike the usual financial accelerator mechanisms that are more concerned with credit availability, the mechanism I present is rooted in the firm-level financing/spending process; it functions even if credit is readily available and asymmetrically affects the business cycle.

Chapter 5 presents an alternative way to consider the financial acceleration due to changes in the credit market. Instead of focusing on the impact of collateral value fluctuations on the ability of borrowers to raise financing, this chapter focuses on generating endogenous defaults emanating from the fluctuations in the borrowers' revenues. If the revenues of the borrowing firms are countercyclical, then default rates and credit spreads would be countercyclical too. There is a rich literature proposing ways to generate endogenous firm bankruptcies. Carlstrom and Fuerst (1997) build on the lending agency costs model by Bernanke and Gertler (1989) and design a general equilibrium model with endogenous defaults, where the reliance of borrowers on their net-worth to secure credit generates a hump-shaped reaction of investments and output to changes in TFP. However, as explained by Gomes, Yaron, and Zhang (2003), this model fails to produce countercyclical default premiums. Pesaran and Xu (2016) use a default mechanism similar to the one I present in chapter 5. A significant difference resides in the way the two authors deal with employment. While Pesaran and Xu choose a specific consumer utility function to disentangle the problem of default from that of labour, I choose to dissociate defaults from labour by assuming that some firms borrow to produce an intermediate good (intermediate producers) and others hire labour and acquire capital to produce the final good (final good production firms). The borrowing and thus bankruptcies are limited to the intermediate producers that transform the economy's single good into an intermediate good without using labour. Other differences include the modelling of the banking sector, a mere multiplier in Pesaran and Xu (2016) while it is assumed to play an important role in the pricing of debt contracts in the model of chapter 5. Finally, default rates end up being invariant in Pesaran and Xu (2016) while they are countercyclical in the model I propose. Christiano, Motto, and Rostagno (2010) develop a large New Keynesian DSGE model with a mechanism for endogenous defaults. The default mechanism they use assumes that an exogenous time-varying process drives the credit riskiness of debt contracts. This process represents the variance of the idiosyncratic shocks affecting the ability of a class of borrowing entrepreneurs to transform capital. The variation of credit risk through time generates countercyclical default rates and credit spreads as long as an empirically reasonable covariance between the credit risk process and aggregate TFP is assumed. In chapter 5, all the model variables' fluctuations are derived from the fluctuation of aggregate TFP. Countercyclical default rates and credit spreads are generated without the need for an ad hoc exogenous stochastic process driving credit risk.

Another strain of literature produces countercyclical credit premiums through lender/borrower relationships. Aliaga-Diaz and Olivero (2010) and Aksoy, Basso, and Coto-Martinez (2013) argue that banks exploit existing lending relationships and the preference of borrowers to stay with the same lender and charge higher credit spreads during slowdowns. This is achieved by using the deep habits framework in Ravn, Schmitt-Groh, and Uribe (2006) to model the costs of bank switching.

By providing a motivation for the firm to cut spending despite abundant liquidity and to under-react to interest rates cuts near the ZLB, the underspending mechanism studied in chapter 2 can be described as a firm based liquidity trap. This connects the thesis to the literature explaining the slow recovery after the GFC through liquidity traps. Eggertsson and Krugman (2012) argue that following a "Minsky moment" where agents realise that debt levels are too high, an aggressive deleveraging cycle begins. This decreases prices and triggers a Fisher debt-deflation cycle. Bacchetta, Benhima, and Kalantzis (2019) build a monetary model with assets scarcity and use it to show that a liquidity trap caused by a persistent deleveraging shock increases real cash holdings and decreases investment and output in the medium term. The authors argue that quantitative easing can lead to a deeper liquidity trap, while a higher government debt can ease assets' scarcity, helping to exit the liquidity trap, but may harm investment in the medium term.

There is a vast literature dealing with the idiosyncratic origins of aggregate fluctuations. The seminal work of Leontief (1941) has pioneered the use of input-output tables to study the effect of industry-specific shocks on aggregate output. In more recent work, Gabaix (2011) argues that idiosyncratic firm-level shocks can explain an important part of the aggregate movement if the firm size distribution presents a fat tail towards large firms. In the same article, the author explains that the fat-tailed firm size distribution has strong empirical justification and that as a result, "the idiosyncratic movements of the largest 100 firms in the United States appear to justify about a third of variations in output growth". Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) argue that even in the case of a very disaggregated economy, i.e. an economy with a large number of industries, the properties of the input-output matrix can guarantee that industry-specific shocks carry on to affect the aggregate quantities and do not vanish due to the law of large numbers. According to the authors, this is the case when some sectors play an asymmetric role as a supplier to other sectors, as in a "star" production network. The nature of the production network linking industries plays a similar role in chapter 3, as I find that a star network makes the asymmetries generated by the chapter 2's main mechanism subsist even when the economy contains a very large number of industries that are subject to independent productivity shocks.

Chapter 2

Macroeconomic Effects of Firms' Underspending in Times of Abundant Credit

2.1 Introduction

This chapter studies a mechanism whereby firms set their production spending at a lower level than the one permitted by the financial resources at their disposal. In the studied framework, underspending happens following unpredicted adverse shocks that affect the profit rates in such a way that distributing money back to investors becomes more appealing to firms than investing in the production process.

In the chapter's setup, money is used by the firm as a store of value to fund future production. However, the firm may decide to spend less than the money originally earmarked for production and distribute the excess cash to shareholders if production unexpectedly becomes less profitable. In real terms, money deteriorates in value with inflation, but this is less of an issue when inflation is low for an extended period of time while uncertainty regarding the performance of other riskier investments is high. Whatever the level of inflation, as one of the safest ways to store value, money provides a lower limit on the return of viable investments from the perspective of rational, profit maximising agents. At the financing stage, firms target higher returns on investment than those offered by simply keeping money in their cash accounts. Usually, the gross nominal returns on investment targeted by firms are higher than the return provided by cash -i.e. one- as these returns need to be large enough to cover the firm's cost of financing. Investors who finance the firm activity (shares and debt holders) also expect a positive return on investment. Investor rewards, seen as a financing cost from the firm perspective, represents a lower bound to the expected return on the projects the firm is undertaking for these projects to be considered worthwhile by a rational firm manager maximising the firm's profit. Following unexpected drops in productivity between the financing stage and the time of spending, the firm can decide to spend less than the previously set financing. This does not occur when productivity matches or beats expectations. The studied mechanism asymmetrically affects the business cycle: it results in the slumps of the cycle becoming more severe and does not operate following shocks that improve productivity.

To clarify ideas, assume a firm operates in a competitive market with free entry and faces the gross financing cost r_t at time t. In addition, assume that its gross return from production $\mathscr{R}_T$ is affected by some uncertainty that remains unresolved until the time the producing starts T. Given the perfect competition and free entry assumptions, the firm makes no profit and no loss on average. At the financing stage t, a rational profit maximising firm would then target an expected return equal to its financing cost [1]

$$\mathbb{E}_t[\mathscr{R}_T] = r_t > 1. \tag{2.1.1}$$

After raising financing, the firm compares the return on producing against the return on holding cash. Barring a large enough unexpected drop between the financing and production stages, the production return $\mathscr{R}_T$ surpasses the return from distributing cash back to shareholders (i.e., 1). If the firm's expected return from producing drops low enough between the financing stage and production time, the firm returns may become lower than that correspond-

[1] As it will be clear below, this depends on the decreasing return to scale assumption of the firm's technology.

ing to keeping cash and not producing

$$\mathscr{R}_T < 1. \tag{2.1.2}$$

The firm would then cancel part of its previous spending plans and enters a low spending mode simply because, at the margin, cash as a store of value outperforms investing in the production process. A first important observation is that, as long as net interest rates remain positive, the mechanism described here only functions following an unexpected drop in expected profits. If firms' returns from producing beat expectations, cash cannot represent a better alternative than spending to produce.

Given the relationship between the firm's expected gross return at the financing stage and the cost of financing, one can rewrite the condition 2.1.2 for firms to enter an underspending mode as follows

$$\mathscr{R}_T - \mathbb{E}_t[\mathscr{R}_T] < -(r_t - 1). \tag{2.1.3}$$

This condition is one of the central results of this chapter. It states that the drop in expected returns required for firms to cancel previous spending plans is tied to the financing costs facing the firm at the time of making the original spending plans. An immediate consequence of this is that firms are more likely to enter an underspending mode by preferring cash distributions to production spending when: uncertainty over the firm's returns from investing in the production process is high, or financing costs are low. Both are features of advanced economies in the period that followed the 2008 Great Recession.

I study this underspending mechanism in the context of a monetary general equilibrium model by building on the work of Christiano and Eichenbaum (1995). In the adopted framework, demand for money is two-fold. First, the firm is subject to a working capital constraint, which means that it needs to secure the funds necessary to pay wages and capital costs before starting production. These funds are assumed to be secured through loans issued by a representative financial intermediary. Second, households are assumed to derive utility from holding money through a Money in the Utility function (MIU) specification. The second assumption helps capture the usefulness of money balances in facilitating transactions.

As is standard in monetised business cycle models, the fluctuations of output are, in part, driven by an exogenous aggregate productivity process, the main innovation being that the financing problem facing the firm is considered separately from its spending problem. This separation gives the firm's management a more realistic economic agency as they exert their control over the cash raised at the financing stage. The firm decides its level of financing first. In order to decide its financing demand, the firm maximises expected profits, considering the prevailing financing costs and the expected returns from future production. Once financing is secured, the level of financing serves as a cap for future production spending. The spending constraint binds as long as productivity does not drop below expectations more than by an amount related to the financing costs in a similar fashion to what equation 2.1.3 indicates. The lower the nominal financing cost, the more likely it is for firms to underspend. More intuitions summarised by equation 2.1.3 are maintained in the context of the studied monetised real business cycle model. The underspending mechanism asymmetrically affects the business cycle: it worsens the slumps of the cycle and does not operate following shocks that improve productivity.

The mechanism studied in this chapter can contribute towards understanding the significant drop in output in 2008. However, given the levels of nominal interest rates facing firms before 2008, a large drop in productivity is required for firms to enter an underspending mode that would worsen the cycle's trough. Fernald (2014) has documented an important decline in total factor productivity (TFP) growth between 2005 and 2008. More recent productivity data published by the Bureau of Labour Statistics indicate a decline in utilisation-adjusted TFP growth after the recession. Other factors contributing to an unexpected deterioration in firms' profits can achieve a similar effect and push firms to spend less than the previously raised financing. For example, a sudden collapse of consumer demand following a deterioration in the households' net worth as in Mian and Sufi (2012) can play a similar role to unexpected drops in productivity.

Related Literature.— This paper explores a firm based financial mechanism while abstracting from the issues related to the inability of agents to raise debt as a result of the worsening in the value of the asset they use as collateral (Kiyotaki and Moore (1997), Bernanke and Gertler (1989) and others). Many authors discussed the role of financial frictions linking lower credit access to the deterioration in the value of collateral in the context of the great recession (e.g. Hall (2011) and Mian and Sufi (2012)). I view this work as complementary to this literature as I explore the consequences of a large sudden drop in TFP on the behaviour of firms that do not suffer from a credit constraint.

I assume that firms have to finance the production costs before engaging in production. This hypothesis is similar to what is assumed in the working capital models such as in Christiano and Eichenbaum (1992) and Cooley and Quadrini (2006). I also draw on this literature when setting the role of unexpected monetary injections: the central bank injects new money into the financial intermediaries to increase the supply of loans available to the production sector. The increase in the money supply is not distributed equally to all agents, which guarantees a role for money in the fluctuations of the model's real variables. However, I depart from this literature when setting the demand for money. Instead of using a cash in advance constraint (CIAC) to generate a demand for cash from households, I adopt a Money In the Utility function approach (MIU) to provide households with a reason to hold cash. The household demand is complemented by the demand for cash emanating from the firms' working capital constraints. The approach I adopt for modelling money provides a more realistic form for the demand for money by households and unlike CIA approaches, does not prevent the underspending mechanism from operating by muting the fluctuations of the firms' revenues.

The studied mechanism provides a potential motivation for the firm to cut spending despite abundant credit by considering the role of money as a store of value. Although hitting the interest rates Zero Lower Bound (ZLB) is not important to the model's functioning, the likelihood of firms underinvesting the cash available to them in the production process is stronger when nominal interest rates are low. The underspending mechanism studied here can be loosely described as a firm based liquidity trap. This connects this work to the literature explaining the slow recovery after the GFC through liquidity traps such as in Guerrieri and Lorenzoni (2017) where the authors build a Bewley-Aiyagari-Hugget type model with households that are subject to a reduction in their borrowing limit. Constrained consumers are then forced to reduce debt, and unconstrained consumers have an incentive to increase their precautionary savings. This creates a powerful downward pressure on interest rates. Interest rates decrease sharply as a result, and this pushes the economy into a liquidity trap. Eggertsson and Krugman (2012) argue that following a "Minsky moment" where agents realise that debt levels are too high, an aggressive deleveraging cycle begins. This decreases prices and triggers a Fisher debt-deflation cycle. Bacchetta, Benhima, and Kalantzis (2019) build a monetary model with assets scarcity and use it to show that a liquidity trap caused by a persistent deleveraging shock increases real cash holdings and decreases investment and output in the medium term. The authors argue that quantitative easing can lead to a deeper liquidity trap, while a higher government debt can ease assets' scarcity, helping to exit the liquidity trap, but may harm investment in the medium term. While the underspending mechanism I study becomes stronger when interest rates are low, hitting the lower zero bound is not crucial for its functioning. The same applies to credit constraints that are often assumed in liquidity trap models to limit the borrower's demand for financing. In the model presented here, I do not assume any form of credit rationing and still establish situations where agents prefer cash to real investments. This is achieved by considering the faith of the cash transferred to the borrowing firm when productivity unexpectedly deteriorates between the time financing is raised and the time of production.

The underspending mechanism studied here functions when the firms' spending upper constraint imposed by the previously set financing is no longer binding. This model feature links this chapter to the numerical literature studying the simulation of dynamic macroeconomic models in the presence of occasionally binding constraints. To deal with this numerical issue, I use the toolkit developed by Guerrieri and Iacoviello (2015). This toolkit (OccBin) adapts a first-order perturbation approach and applies it in a piecewise fashion to solve dynamic models with occasionally binding constraints.

Finally, there is the link with the cash hoarding literature (e.g. Opler, Pinkowitz, Stulz, and Williamson (1999)). The literature describes two main benefits of hoarding cash. The first concerns transaction costs savings and the avoidance of having to rely on liquidating assets at a cost to finance new investment opportunities (transaction costs motive). The second is related to the possibility of financing becoming unavailable or excessively costly in the future when investment opportunities materialise (precautionary savings motive). The motive of transaction costs cannot be analysed in this chapter because the firm is assumed to be very short-lived and does not hold any assets besides cash. In the model studied in this chapter, the firm is subject to a working capital constraint, implying that ad hoc financing is unavailable at the time of production. The firm raises all its financing needs before the start of production and cannot increase its financing just before production starts. This is akin to an extreme precautionary saving motive. However, the fact that the firm is very short-lived means that it cannot hoard the extra cash. The firm distributes the idle cash back to shareholders immediately.

The model is presented in section 2.2. Section 3.5 presents and comments the simulation results and section 2.4 concludes.

2.2 The model

In this section, I build a monetised real business cycle model that illustrates the underspending mechanism described in the introduction. In order to study the incentive of the firm to underspend, I separate the firm's financing problem from its spending problem. The firm first sets its financing based on its assessment of productivity at the financing stage. Production spending is decided some time after the financing stage and the previously set financing acts as an upper limit for potential spending. Following some unpredicted deterioration in productivity between the financing stage and spending stage, the firm can set spending at a lower level than the financing constraint.

In the set-up I consider, households maximize their utility to decide consumption and leisure subject to a budget constraint. The single consumption good is produced by firms that are constrained by a Cobb-Douglas production function. Production is financed by households through bond issuance and in the absence of credit risk, all firms face the same interest rate:

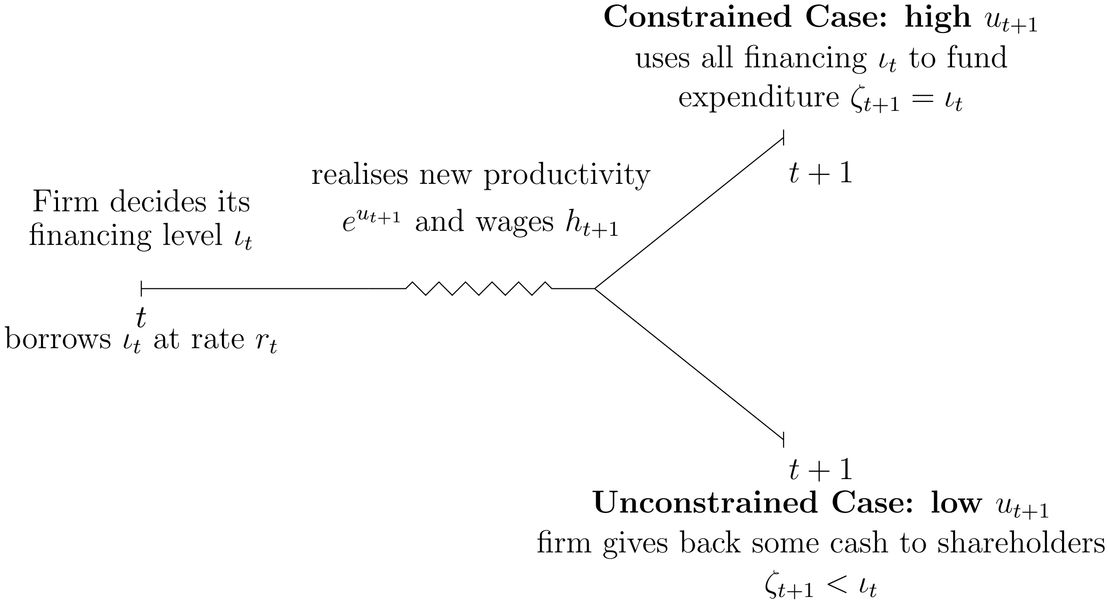

Figure 2.1: Timeline of the firm financing and spending process.

r_t. The diagram in figure 2.1 explains the firm's financing/spending decision process: the representative firm decides its financing level ι_t at time t and raises the required amount using bonds before discovering the new productivity $e^{u_{t+1}}$ between time t and time $t+1$. At time $t+1$, the profit-maximizing firm assesses its own productivity and the prevailing wages then chooses whether to spend all the raised financing ι_t (constrained spending case) or to spend less than the raised financing level $\zeta_{t+1} < \iota_t$ (unconstrained spending case).

Firms are subject to a working capital constraint and money is introduced to satisfy the firms' need to hold cash before production starts. In addition, households are supposed to derive utility from holding money. The fluctuation of the money stock are controlled by a single monetary authority that may inject money in the economy to ease the credit conditions facing firms as in Christiano and Eichenbaum (1992). The remainder of this section details the behaviours of households, firms and the monetary authority before commenting on the level of productivity drop required for firms to enter an underspending mode.

2.2.1 Households

Households derive utility from consumption, leisure and real money holdings. Their utility function takes the form

$$\mathscr{U}_t = \ln(c_t) + \frac{\psi}{1-\nu}\left(\frac{M_t^H}{P_t}\right)^{1-\nu} - \frac{\chi}{1+\eta} l_t^{1+\eta}, \tag{2.2.1}$$

where l_t denotes the household's labour, c_t consumption, M_t^H the households' money holdings, P_t the price of the consumption good, ψ is a paramter that helps control the level of real money holdings by households, ν is the curvature on the utility form holding real cash balances, η the curvature on dislike of labour and χ is a parameter that controls for households' dislike of labour. The direct demand by households for real money balances can be justified by a role played by money to simplify transactions.[2] Consumers maximise their expected lifetime utility discounted at rate β under their budget constraint in order to set their consumption, their leisure time $1 - l_t$, their savings through real capital k_t, their real money holdings $m_t^H := M_t^H / P_t$ and how much they save through (real) deposits b_t

$$\max_{c_s, l_s, b_s, m_s^H} \mathbf{E}_t \sum_{s=t}^{\infty} \beta^s \mathscr{U}_s, \tag{2.2.2}$$

subject to the sequence budget constraint

$$m_s^H + c_s + k_s + b_s \leq \frac{m_{s-1}^H}{1+\pi_s} + (1-\delta)k_{s-1} + r_s^K k_{s-1} + r_{s-1}\frac{b_{s-1}}{1+\pi_s} + \omega_s l_s + \Pi_s^F + \Pi_s^B, \tag{2.2.3}$$

Where w is the real wage, r the gross nominal interest rate on deposits and r^K the real net rate of return on capital. The terms Π^F and Π^B represent the real profits distributed respectively by the firms and the banks, both owned by households. Equity investors (i.e., households) are assumed to cover the debt payment shortfall in the case where the firm's proceeds do not cover its debt obligations. This means that the real profit Π^F provided by the firm to households can be negative to help avoid firms' bankruptcies.

[2] See Croushore (1993) on the equivalence between money in the utility function and a shopping-time model.

Solving the household's optimisation problem, the nominal interest rate r_t and the (real) cost of renting capital r_t^K satisfy the Euler equation

$$1 = \beta r_t \mathbf{E}_t \frac{c_t}{(1+\pi_{t+1})c_{t+1}}, \tag{2.2.4}$$

$$1 = \beta \mathbf{E}_t r_{t+1}^K \frac{c_t}{c_{t+1}} + (1-\delta)\frac{c_t}{c_{t+1}}. \tag{2.2.5}$$

In addition, the households' problem yields the labour supply condition

$$\chi c_t = \omega_t l_t^{-\eta} \tag{2.2.6}$$

and the demand for real money

$$(m_t^H)^\nu = \psi \frac{r_t}{r_t - 1} c_t. \tag{2.2.7}$$

2.2.2 Financial intermediation and monetary policy

Following Christiano and Eichenbaum (1992), a direct monetary channel is operated by injecting money in financial intermediaries to help finance the loans extended to production firms. The representative bank provides the firms with the loan ι_t charging the gross nominal rate r_t^F. The bank finances its loans operations using households deposits b_t and a monetary injection from the central bank that is proportional to the existing nominal stock of money

$$\iota_t = b_t + x_t^B \frac{m_{t-1}}{1+\pi_t}, \tag{2.2.8}$$

where m_{t-1} is the existing total real stock of money inherited from the previous time period and x_t^B is the relative change in the nominal stock of money. The bank profit function is then

$$\Pi_{t+1}^B = r_t^F \iota_t - r_t b_t. \tag{2.2.9}$$

Or equivalently

$$\Pi^B_{t+1} = r^F_t x^B_t \frac{m_{t-1}}{1+\pi_t} + (r^F_t - r_t)b_t. \tag{2.2.10}$$

The monetary authority provides the financial intermediaries with the new money in the goal of easing credit conditions for firms. This prevents the increase in the money supply from being distributed equally to all agents and guarantees a role for money in the model's fluctuations. Following Christiano and Eichenbaum (1995), I assume that the bank's profit is zero ($\Pi^B_{t+1} = 0$). This yields

$$r^F_t \iota_t = r_t b_t. \tag{2.2.11}$$

by controlling the injection x^B_t helps control the lending rate

$$r^F_t = \frac{1}{1 + x^B_t.m_{t-1}/(b_t(1+\pi_t))} r_t. \tag{2.2.12}$$

I assume that the money growth x^B_t follows the AR(1) process

$$x^B_t = (1-\rho_B)x^B + \rho_B x^B_{t-1} + \sigma_B v^B_t, \tag{2.2.13}$$

where x^B is the steady-state value of x^B_t, σ_B is the volatility of innovations, ρ_B is the autocorrelation parameter and v^B_t is an $i.i.d$ error term that is independent of all other shocks in the model.

The assumption that money is directly injected in the financial intermediaries can be justified by the the central bank's ability of to affect lending through the use of open-market operations (Christiano and Eichenbaum (1992)). All the stock of money available at the end of the production cycle is inherited by the households and is either kept in real cash balances or deposited with the financial intermediary

$$\frac{m_{t-1}}{1+\pi_t} = m^H_t + b_t. \tag{2.2.14}$$

The latter money clearing condition, combined with the household demand for money emanating from utility maximisation as described by equation 2.2.1, helps determine prices in the model.

$$\frac{m_{t-1}}{1+\pi_t} = \left(\psi \frac{r_t}{r_t - 1} y_t\right)^{1/\nu} + b_t. \tag{2.2.15}$$

It is important to note that unlike in Christiano and Eichenbaum (1992), the household decisions in terms of consumption, money holdings and deposits are not made before the injection of new money by the monetary authority. While the new money is not distributed equally to all agents, it still influences the decisions of households; for instance, through increasing inflation, the monetary injection increases the nominal rate r_t and pushes households to save more through deposits at the expense of holding money.

Further to the direct monetary injection into financial intermediaries, I assume that money holding is changes every period in a way that equally affects all agents. This is achieved by distributing the new money to households at the start of every period. Overall, money grows because of the accumulated distributions to banks and to households.

$$m_t = \frac{m_{t-1}}{1+\pi_t} \left(x_t^B + x_t^H\right), \tag{2.2.16}$$

Where x_t^H also follows an AR(1) process

$$x_t^H = (1-\rho_H)x^H + \rho_H x_{t-1}^H + \sigma_H v_t^H, \tag{2.2.17}$$

with x^H being the steady-state value of x_t^H, σ_H is the volatility of innovations, ρ_H is the autocorrelation parameter and v_t^H is an $i.i.d$ error term that is independent of all other shocks in the model. While the injection of money into the financial intermediaries directly affects both inflation the lending rate, the change of the money stock through x_t^H only directly affects inflation.

2.2.3 Firms

I assume the existence of a production sector with a representative firm operating the technology

$$y_t = e^{u_t} \left(k_{t-1}^{\alpha} l_t^{1-\alpha} \right)^{\gamma}, \tag{2.2.18}$$

where k_t denotes capital and l_t the labour it uses, α is the share of capital, γ a return to scale parameter and e^{u_t} a stochastic productivity process. Within the studied framework, increasing returns to scale would imply infinite financing demand and constant return to scale would lead to undetermined levels of firm financing. Empirically, many studies could not reject the constant return to scale hypothesis on the industry level while others point towards a slightly decreasing returns to scale.[3] Decreasing returns to scale are therefore used as a source of curvature in the profit function that guarantees a unique solution to the financing problem ($\gamma < 1$).[4]

The aggregate log-productivity u_t is assumed to follow an $AR(1)$ process with the volatility parameter σ_u and mean-reversion ρ_u.

$$u_t = \rho_u u_{t-1} + \sigma_u e_t.$$

The representative firm finances production using loan contracts that are issued by financial intermediaries at a gross rate r_t^F. At time t, the firm decides the nominal financing amount $P_t \iota_t$ that will potentially be invested in the next period's production process (ι_t denotes real financing and P_t the price of the consumed good). To do so, the firm maximises its expected real profit

$$\max_{\iota_t} \mathbf{E}_t P_{t+1} \Pi_{t+1}^F. \tag{2.2.19}$$

The firm's real profit is a function of future sales proceeds y_{t+1}, the real gross cost of financing

[3]Syverson (2004), Olley and Pakes (1996) and others could not statistically reject the constant return to scale hypothesis. Other studies find slight to moderate decreasing return to scale, for example, Gao and Kehrig (2017).

[4]This is consistent with the literature concerned with heterogeneous firms as for or example in Restuccia and Rogerson (2008) and Bartelsman, Haltiwanger, and Scarpetta (2013).

$r_t^F \frac{\iota_t}{1+\pi_{t+1}}$ and the real unspent financing $\frac{\iota_t}{1+\pi_{t+1}} - \omega_{t+1} l_{t+1} - r_{t+1}^K k_t$

$$\Pi_{t+1}^F = y_{t+1} - r_t^F \frac{\iota_t}{1+\pi_{t+1}} + \left\{ \frac{\iota_t}{1+\pi_{t+1}} - \omega_{t+1} l_{t+1} - r_{t+1}^K k_t \right\} \tag{2.2.20}$$

The unspent financing term $\left\{ \frac{\iota_t}{1+\pi_{t+1}} - \omega_{t+1} l_{t+1} - r_{t+1}^K k_t \right\}$ is nil when the firm is constrained at the spending stage and is positive otherwise. Note that no discounting of the profit is needed because all the cash-flows of the firm happen at time $t+1$. As long as the discounting rate is known at time t (which I assume), discounting plays no role in the firm's financing problem.[5]

At period $t+1$, the representative firm chooses labour l_{t+1} and rented labour k_{t+1} to maximise profit with spending being constrained by the previously set level of financing $\iota_t/(1+\pi_{t+1})$ corrected for inflation. The terms ι_t and $r_t \iota_t$ in the profit function expression 2.2.20 are predetermined at time t, so the profit maximisation simplifies to

$$\max_{l_{t+1}, k_{t+1}} y_{t+1} - \omega_{t+1} l_{t+1} - r_{t+1}^K k_t, \tag{2.2.21}$$

$$\text{s.t. } \omega_{t+1} l_{t+1} + r_{t+1}^K k_t \leq \frac{\iota_t}{1+\pi_{t+1}}. \tag{2.2.22}$$

Increasing returns to scale being excluded ($\gamma < 1$), it is a priori not obvious whether the constraint 2.2.22 holds or not. Let us define ζ_{t+1} as the cost of production $t+1$

$$\zeta_{t+1} := \omega_{t+1} l_t + r_{t+1}^K k_t. \tag{2.2.23}$$

From the first order conditions of the spending problem 2.2.21

$$r_{t+1}^K k_t = \alpha \zeta_{t+1} \tag{2.2.24}$$

$$\omega_{t+1} l_{t+1} = (1-\alpha) \zeta_{t+1} \tag{2.2.25}$$

Exploiting the first order conditions above, one can rewrite production as a function of the

[5] A common practice is to assume that the firm uses the real discount rate $(1/r_t)\mathbf{E}_t(1+\pi_{t+1})$.

actual expenditure level ζ_{t+1}, wages and the stochastic productivity[6]

$$y_{t+1} = e^{u_{t+1}} \frac{\zeta_{t+1}^{\gamma}}{\lambda_{t+1}^{\gamma}}, \tag{2.2.26}$$

where λ_{t+1} represents the marginal cost of production at time $t+1$.

$$\lambda_{t+1} = \frac{(r_{t+1}^{K})^{\alpha} \omega_{t+1}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{2.2.27}$$

The firm's financing problem simplifies to the problem below, where the firm has to decide its level of real spending given the previously set financing constraint

$$\max_{\zeta_{t+1}} e^{u_{t+1}} \frac{\zeta_{t+1}^{\gamma}}{\lambda_{t+1}^{\gamma}} - \zeta_{t+1}, \tag{2.2.28}$$

$$\text{s.a. } \zeta_{t+1} \leq \frac{\iota_t}{1+\pi_{t+1}}. \tag{2.2.29}$$

If the financing constraint is not binding ($\zeta_{t+1} < \frac{\iota_t}{1+\pi_{t+1}}$), the firm sets labour such as the marginal return on spending is one, the same as the marginal return on cash

$$\gamma e^{u_{t+1}} \frac{\zeta_{t+1}^{\gamma-1}}{\lambda_{t+1}^{\gamma}} = 1. \tag{2.2.30}$$

Or equivalently,

$$\zeta_{t+1} = \gamma y_{t+1}. \tag{2.2.31}$$

In other words, when unconstrained by financing, the representative firm sets its expenditure at a level where it is indifferent between producing and simply distributing cash to shareholders. The condition 2.2.30 yields the following inequality verified by underspending firms

$$\gamma e^{u_{t+1}} \frac{\iota_t^{\gamma-1}}{(1+\pi_{t+1})^{\gamma-1}\lambda_{t+1}^{\gamma}} \leq 1. \tag{2.2.32}$$

[6]This formulation of production justifies the decreasing returns to scale assumption. The firm profit function at time $t+1$ is $e^{u_{t+1}} \frac{\zeta_{t+1}^{\gamma}}{(\lambda_{t+1})^{\gamma}} - \zeta_{t+1}$. Clearly, if $\gamma = 1$ spending can be undetermined and if $\gamma > 1$, firms would prefer to spend an infinite amount and the financing demand would be infinite as a result.

The latter condition stipulates that, for unconstrained firms, the marginal return on spending at the constraint is less than the marginal return of distributing cash to shareholders. This underspending characterisation can be rewritten as follows

$$u_{t+1} \leq \xi_{t+1}, \tag{2.2.33}$$

where the cutting point $\xi_{t+1}(\epsilon)$ is defined as

$$\xi_{t+1} = \ln\left(\frac{\lambda_{t+1}^{\gamma}\iota_t^{1-\gamma}}{\gamma(1+\pi_{t+1})^{1-\gamma}}\right). \tag{2.2.34}$$

Everything else being equal, underspending is more (less) likely when u_{t+1} is lower (higher), the previously set real financing ι_t is higher (lower), the real marginal cost of production λ_{t+1} are higher (lower) and inflation π_{t+1} is lower (higher). The formulae below summarise the results established so far regarding spending and production

$$\zeta_{t+1} = \min\left\{\frac{\iota_t}{1+\pi_{t+1}}, \left(\gamma\frac{e^{u_{t+1}}}{\lambda_{t+1}^{\gamma}}\right)^{1/(1-\gamma)}\right\}, \tag{2.2.35}$$

$$y_{t+1} = \min\left\{\frac{e^{u_{t+1}}\iota_t^{\gamma}}{(1+\pi_{t+1})^{\gamma}\lambda_{t+1}^{\gamma}}, \left(\gamma^{\gamma}\frac{e^{u_{t+1}}}{\lambda_{t+1}^{\gamma}}\right)^{1/(1-\gamma)}\right\}. \tag{2.2.36}$$

One can rewrite the last two equations by exploiting the definition of ξ_{t+1}

$$\zeta_{t+1} = \left(\gamma\frac{e^{u_{t+1}}}{\lambda_{t+1}^{\gamma}}\right)^{1/(1-\gamma)} \min\left\{1, e^{(\xi_{t+1}-u_{t+1})/(1-\gamma)}\right\}, \tag{2.2.37}$$

$$y_{t+1} = \left(\gamma^{\gamma}\frac{e^{u_{t+1}}}{\lambda_{t+1}^{\gamma}}\right)^{1/(1-\gamma)} \min\left\{1, e^{\gamma(\xi_{t+1}-u_{t+1})/(1-\gamma)}\right\}. \tag{2.2.38}$$

Note how inflation plays a direct role in setting production spending and production only if the firm is constrained by previous financing.[7]

The spending and production expressions 2.2.35 and 2.2.36 yield the derivatives of spending

[7] Inflation plays an indirect role in setting the production spending and the production of unconstrained firms through its influence on the cost of producing λ_{t+1}.

and production with regard to debt financing

$$\frac{\partial \zeta_{t+1}}{\partial \iota_t} = \mathbf{1}_{u_{t+1} > \xi_{t+1}} \frac{1}{1+\pi_{t+1}}, \tag{2.2.39}$$

$$\frac{\partial y_{t+1}}{\partial \iota_t} = \mathbf{1}_{u_{t+1} > \xi_{t+1}} \gamma \frac{e^{u_{i,t+1}} \iota_t^{\gamma-1}}{(1+\pi_{t+1})^{\gamma} \lambda_{t+1}^{\gamma}}. \tag{2.2.40}$$

Hence the first order condition emanating from problem financing problem 2.2.19 is given by

$$r_t^F = \mathbf{E}_t \left[\mathbf{1}_{u_{t+1} \leq \xi_{t+1}} + \mathbf{1}_{u_{t+1} > \xi_{t+1}} e^{u_{t+1} - \xi_{t+1}} \right]. \tag{2.2.41}$$

Or, equivalently

$$r_t^F = \mathbf{E}_t \max \left\{ 1, e^{u_{t+1} - \xi_{t+1}} \right\}. \tag{2.2.42}$$

The last financing equation shows that, when underspending by firms is possible, money provides a floor for the marginal return on nominal debt financing $P_t \iota_t$ (the floor being 1). The ability to underspend provides firms with an extra protection pushing them to increase their demand for new financing.

2.2.4 Equilibrium and market clearing

The equilibrium is realised when prices (w_t, r_t^K, r_t, r_t^F) and the quantities c_t, b_t, l_t, k_t, ι_t, ζ_t, l_t and k_t are such that firms maximise expected profit at the financing stage subject to the technology constraint and maximise profit at the spending stage subject to the technology and financing constraints, households maximise utility subject to their budget constraint and the various markets within the economy clear. These markets are:

- The good's market where all the production is either consumed by households or used to accumulate physical capital

$$y_t = c_t + k_t - (1-\delta) k_{t-1}. \tag{2.2.43}$$

- The labour market, where households labour supply must meet firms' labour demand
- The market for real capital, where the capital accumulated by households is used by firms in the following time period
- The loan market where households' supply of deposits and the monetary transfers to banks $x_t^B \frac{m_{t-1}}{1+\pi_t}$ must meet the production sector financing demands

$$b_t + x_t^B \frac{m_{t-1}}{1+\pi_t} = \iota_t. \tag{2.2.44}$$

2.2.5 Critical shock level triggering firms' underspending

As noted above, firms' underspending occurs when the log-TFP u_t shock is lower than ξ_t. The threshold level ξ_t depends on variables determined at time t as well as on previously determined variables. To simplify the study of this threshold, I consider its steady-state value ξ, that is derived from the steady-state loan interest rate by writing the steady-state version of the financing equation 2.2.41

$$r^F = e^{-\xi}\Phi\left(-\frac{\xi}{\sigma_u}\right) + \Phi\left(\frac{\xi}{\sigma_u}\right). \tag{2.2.45}$$

r^F is a decreasing function of the variable ξ when the net lending interest rate is positive ($r^F > 1$). In graph 2.2, I show the threshold ξ as a function of the (quarterly, non-annualised) net lending rate $r^F - 1$. The graph assumes quarterly periods and $\sigma_u = 1\%$, the same value assumed in the section 3.5 dedicated to the calibration and simulation of the model. For all non-negative values of the steady-state net loan rate $r^F - 1$, the critical value ξ is negative, confirming that underspending happens after drops in productivity. Furthermore, underspending is more likely (ξ is higher) when the lending net rate is closer to zero. Assuming a quarterly net rate of lending around 1% implies that log-TFP needs to drop by more than a standard deviation for the firms' underspending to occur.

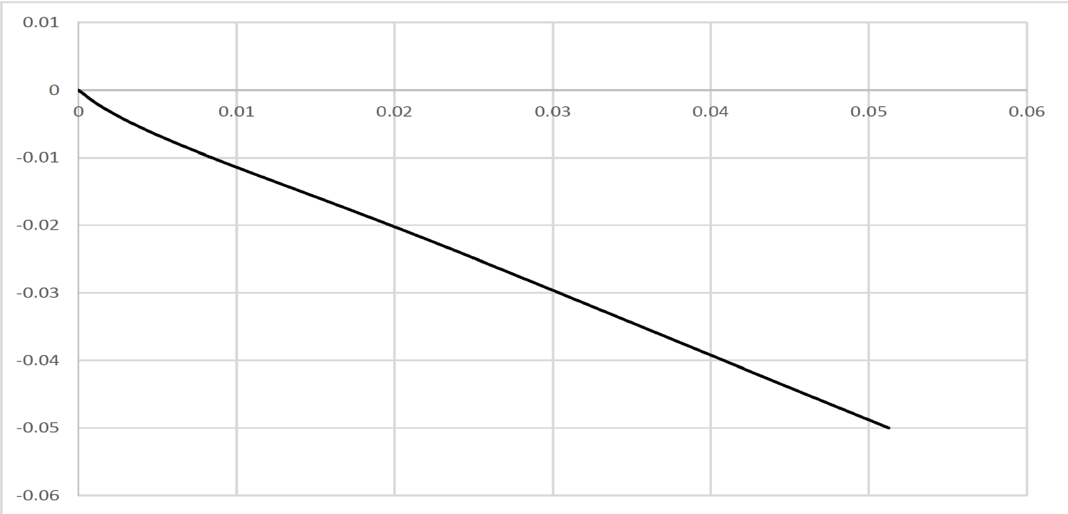

Figure 2.2: Steady-state threshold ξ as a function of the lending rate $r^F - 1$.

2.3 Simulation results

2.3.1 Calibration and steady-state results

The utility discount factor β determines the equilibrium rates of financing and is therefore key to the main mechanism of the model. Mehra and Prescott (1985) report that between 1889 and 1978 the average annual real return on equity was 7%, while the average annual real return on short term debt was 1%. In the current set-up, I assume an investment horizon of one year; I then target an equilibrium interest rate closer to the return on short term debt at 2%, which is more in line with the low real interest rates experienced since 2008. This pins down preference discounting $\beta = 0.995$, given that I assume quarterly periods. I assume no monetary change in the money allocated directly to households in the steady-state $x^H = 0$. The steady-state increase in money stock through injections in financial intermediaries x^B is set to 0.5% to match an annual steady-state inflation at 2%. This implies that in the steady-state banks' lending rates are lower than the deposit rates. Following Atkeson and Kehoe (2005), I assume that the firm return to scale parameter takes the value $\gamma = 0.95$.[8] The volatility

[8]See also Atkenson, Khan, and Ohanian (1996). Importantly, the assumed value of γ does not impact the effects discussed in this section as long as some decreasing return to scale is maintained $\gamma < 1$.

parameter $\sigma = 1\%$ is chosen to match a quarterly productivity fluctuation around 1% and the mean reversion parameter is set at $\rho = 0.8$. The calibration is performed in the steady-state to match a steady-state level of employment of $\bar{l} = 0.33$. This determines the value of the labour provision parameter χ.[9] Furthermore, the parameter ψ is chosen so that the steady-state of money balances to deposits m^H/m matches the M1/M2 ratio for the United States around 25%. The remaining parameters are standard and are borrowed from the literature (table 5.1).

The effect of the preference discounting parameter β on the steady-state variables is presented in figure 2.3. Higher preference discounting implies lower the steady-state real rate deposit rate $1/\beta - 1$, which in turn, implies lower lending rate r^F in the steady-state and higher likelihood of underspending (figure 2.2). Lower steady-state lending rates also imply higher steady-state financing, capital, labour, output and utility. The steady-state effect of lower money increase through direct distribution to households (lower x^H) is qualitatively similar to the effect of higher preference discounting as both lead to lower deposit rates and therefore lower lending rates facing the firms. In the case of lower x^H, the lower nominal deposit rates are a result of lower steady-state inflation (figure 2.4). On the other hand, rising inflation through monetary injections in the financial intermediaries (higher x^B) lowers the nominal lending rates and increases the probability of underspending. Higher values of x^B imply higher steady-state labour, capital, output consumption, and utility through its impact on steady-state lending rates.

The drop in log productivity required for the model's mechanism to operate as a function of the steady-state interest rates is shown in figure 2.2. In order to show the dynamic effects of the underspending mechanism, I assume a $4.5\times$ standard deviations unexpected drop in TFP. The results are presented and commented in the next subsection.

2.3.2 The dynamic effects of the model's mechanism

To deal with the numerical difficulties related to the occasionally binding spending constraint, I use the OccBin toolkit to simulate the model in the presence of the underspending mechanism.

[9]For more details regarding the steady-state equations and the calibration process, please refer to appendix 2.A.2.

OccBin is a toolkit developed by Luca Guerrieri and Matteo Iacoviello that adapts a first-order perturbation approach to solve dynamic models with occasionally binding constraints (Guerrieri and Iacoviello (2015)).

I present impulse response functions illustrating the dynamic effects of the model's main mechanism. Figure 2.6 compares impulse responses of the model with the firm's underspending mechanism to a version of the model where the financing constraints are always binding (benchmark). The figure shows the impulse responses of the main model variables to a large negative shock to productivity ($-4.5\times$ standard deviations).[10] When the model's main mechanism is functioning, the negative productivity shock impacts the model's variables sooner. As firms gain knowledge of the new lower levels of productivity, they adapt by reducing expenditure before the lower financing levels affect the economy. By contrast, when the firms cannot adjust expenditure, their previous level of financing, which was based on a more optimistic view of productivity, helps dampen the severity of the current negative shock. The drop in spending by the unconstrained firms lowers labour demand and implies that wages decrease to lower a level than when spending is always constrained. This implies a larger reaction of labour's provision to the drop in productivity when spending is not constrained and, in turn, implies a more severe output drop. As output drops further, less is invested in forming capital and as a result, real investments drop further . As a store of value, cash provides firms with extra protection in the presence of the underspending mechanism. The TFP shock pushes households to increase their deposit demands, thus bringing deposit rates lower. Lower deposit rates imply lower loan rates, which increases the likelihood of underspending. As underspending becomes more likely, the protection provided by the possibility to underspend later increases the demand of firms for financing further, hence a less severe reaction of firms' financing to negative productivity shocks when firms are allowed to underspend. The moderate reaction of financing in the presence of underspending implies that wages, labour, output and investment react less in the main model in the period following the shock. Furthermore, the moderate reaction of firms' financing implies a more moderate reaction of deposit rates and loan rates in the presence of

[10] As shown in figure 2.2, the steady-state value of ξ decreases in the the steady-state loan rate. In the used calibration, the steady-state loan rate is 3.7%.

underspending. Finally, lower output following the negative productivity shock causes higher inflation in both models as the supply of money does not adjust following the real shock.

Large unpredicted real shocks are required for firms to underspend. Following moderate, unpredicted negative TFP shocks, the underspending mechanism does not function and the immediate reaction of the main model in terms of real wages, labour, output and investments is similar to the reaction of the benchmark model without the underspending mechanism (figure 2.7). However, the option to potentially underspend in the future keep the demand for financing higher in the main model, thus moderating the impact on wages, labour, output and investments in the time period that follows the shock. The fluctuations of financing increases the autocorrelation of output, investment and other aggregates. By moderating the fluctuations of firms' financing, the underspending mechanism reduces the autocorrelation of output, investments and labour.

Following (unpredicted) positive TFP shocks, the underspending mechanism cannot function. The reaction of real aggregates is similar in the main and the benchmark models following positive shocks to TFP, even when these shocks are large (figure 2.8). The model's main mechanism has therefore important implications in terms of cycle asymmetry. The real effects of Firm's underspending can be significant following large unpredicted real shocks but remain muted following positive shocks.

There are two sources of monetary shocks in the model. The process x_t^H that governs new money supplied directly to households before they engage in any saving of or consumption activities and the process x_t^B that govern monetary injections into financial intermediaries. A positive unpredicted shock to the process x_t^H (figure 2.9) creates a pure inflation effect. Higher inflation implies a lower value of the money held by firms, which pushes real wages, labour, output, and investments lower following a positive shock to x_t^H. The behaviour following the pure inflation shock is similar in both the main model and the benchmark model. A positive inflation shock decreases the real value of the money held by firms and cannot cause them to underspend.

A positive shock to the process x_t^B has two effects; an inflation effect because of the increase in money supply and a liquidity effect as more money is available to financial intermediaries to finance the productive sector. Figure 2.10 shows that, similarly to the pure inflation shock, a positive shock to the money injected in financial intermediaries reduces the value of the cash held by firms and pushes real wages, labour, output and investments lower. However, the liquidity effect on the loan rate dominates the inflation effect: the loan rate drops lower following an unpredicted positive shock to the money injected in financial intermediaries. Lower financing rates pushes firms to increase financing demand in both model. This pushes real wages, labour, output and investments higher in the period following the shock. The shock does not trigger the underspending mechanism, so the aggregate variables' reaction is similar in the main and benchmark models.

To summarise, the model's main mechanism amplifies the extreme lows of the business cycle by pushing firms into an underspending mode following large unpredicted shocks to TFP. Underspending has, however, little effect on real aggregates following moderate or positive TFP shocks. Given the current monetary set-up, focused on the liquidity effects of monetary injections, firms' underspending has little effect on the real impact of monetary injections.

2.4 Concluding remarks

I presented a financial mechanism rooted in how firms change their behaviour towards setting current expenditure as a reaction to unexpected large negative shocks to productivity. When affected by large unpridcted drops in productivity, firms can react by reducing spending to lower levels than those permitted by the financial resources at their disposal. The change of spending behaviour can worsen and lengthen the trough of the business cycle, as firms adjust their expenditure before the effects of lower corporate financing hit the economy. This mechanism has important implications in terms of business cycle asymmetry. Firms would only adjust expenditures following large negative shocks to productivity while they remain constrained by previously determined levels of financing if productivity is improving.

I show that firms' underspending is more likely in a low (nominal) interest rates environment.

Model parameter	**Value**
Households preferences	
β discount factor	0.995
η curvature on labour	1
χ disutility of labour	7.87
ν curvature on real money	2
ψ real money demand	0.19%
Technology	
γ return to scale	0.95
α capital share	0.33
ρ aggregate TFP persistence	0.8
σ aggregate TFP volatility	1%
δ depreciation rate of capital	2.5%
Money Supply	
x^H steady-state money supply to households	0%
x^B steady-state money supply to financial intermediaries	0.5%
ρ_m persistence of monetary shocks	80%
σ_m volatility of monetary shocks	0.25%

Table 2.1: Assumed and calibrated model parameters.

This means that when firms' financing rates are low, smaller drops in productivity are required for firms' to enter an underspending mode. Nominal financing rates can be low because of a low inflation environment or through central bank accommodative activity (liquidity effects in the model). Low inflation and expansionary central banks activism are both features of the advanced economies in the post-2008 period.

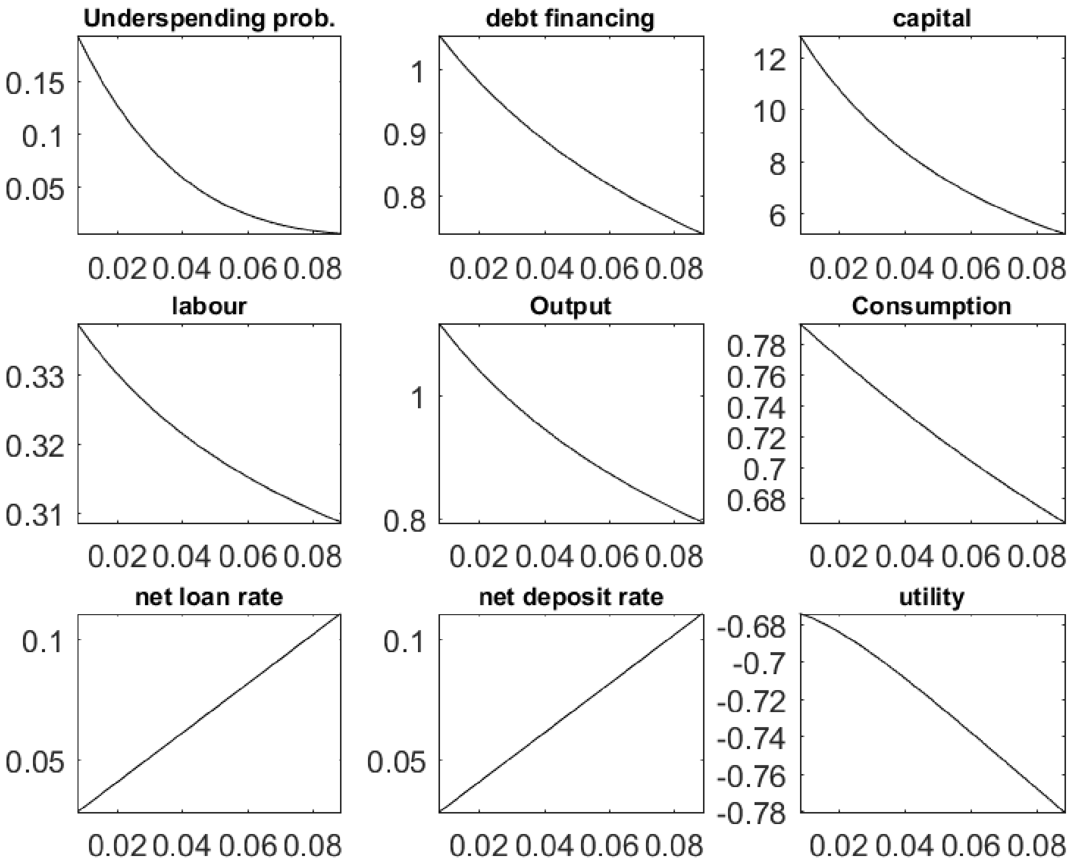

Figure 2.3: Effect of changing utility discounting β on steady-state variables. The figures show the steady-state variables as a function of the steady-state real deposit rate $1/\beta - 1$.

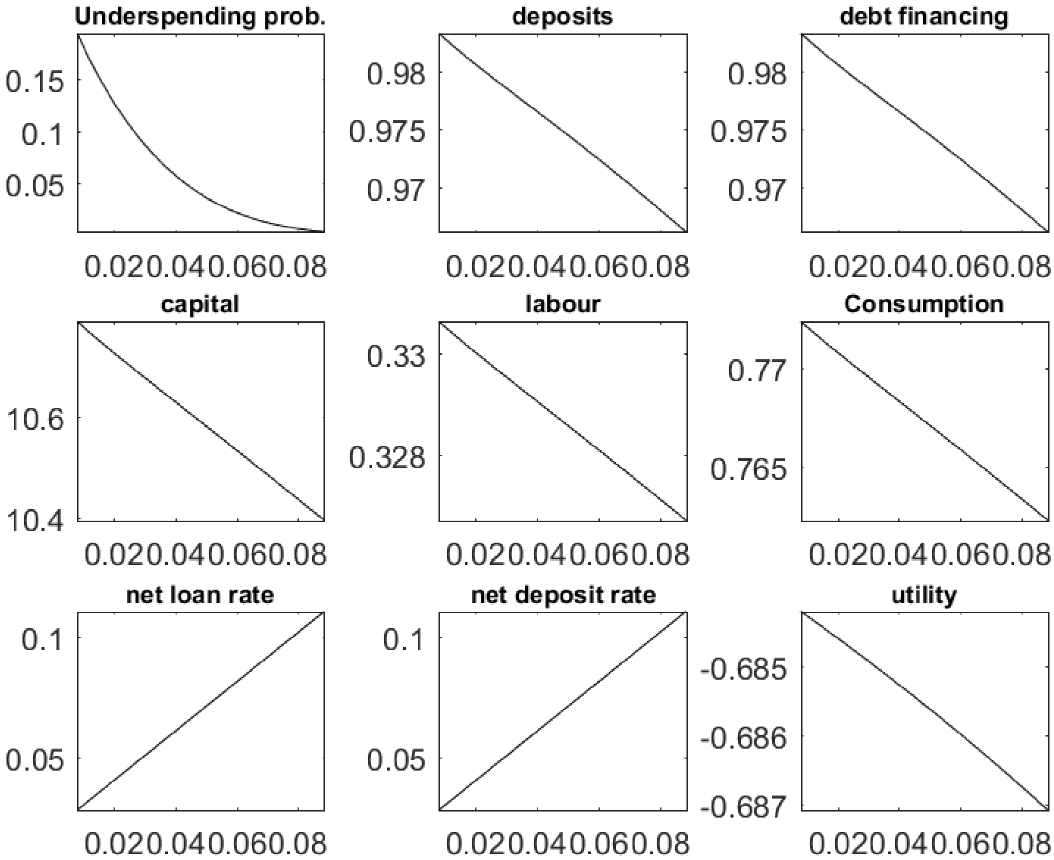

Figure 2.4: Pure inflation effects on steady-state variables. The figures show the steady-state variables as a function of the steady-state growth of money x^H.

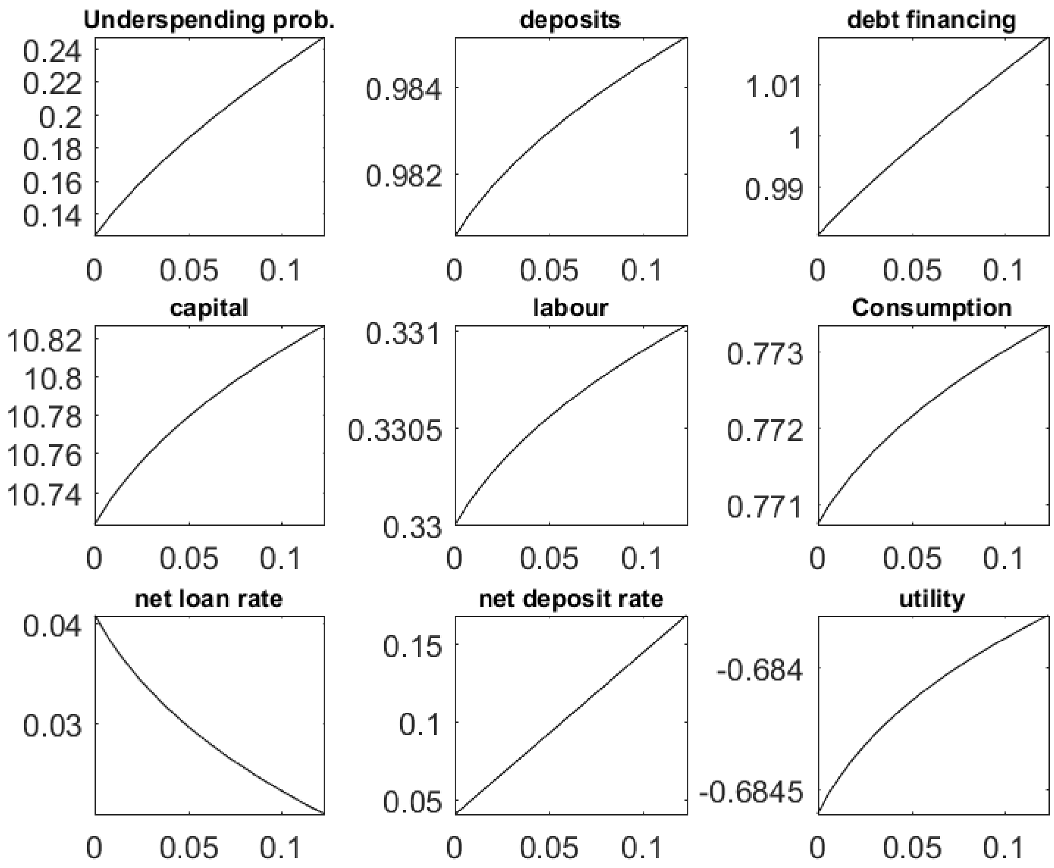

Figure 2.5: The effect of injecting money in the financial intermediaries on the steady-state variables. The figures show the steady-state variables as a function of the steady-state growth of money dedicated to financial intermediaries x^B.

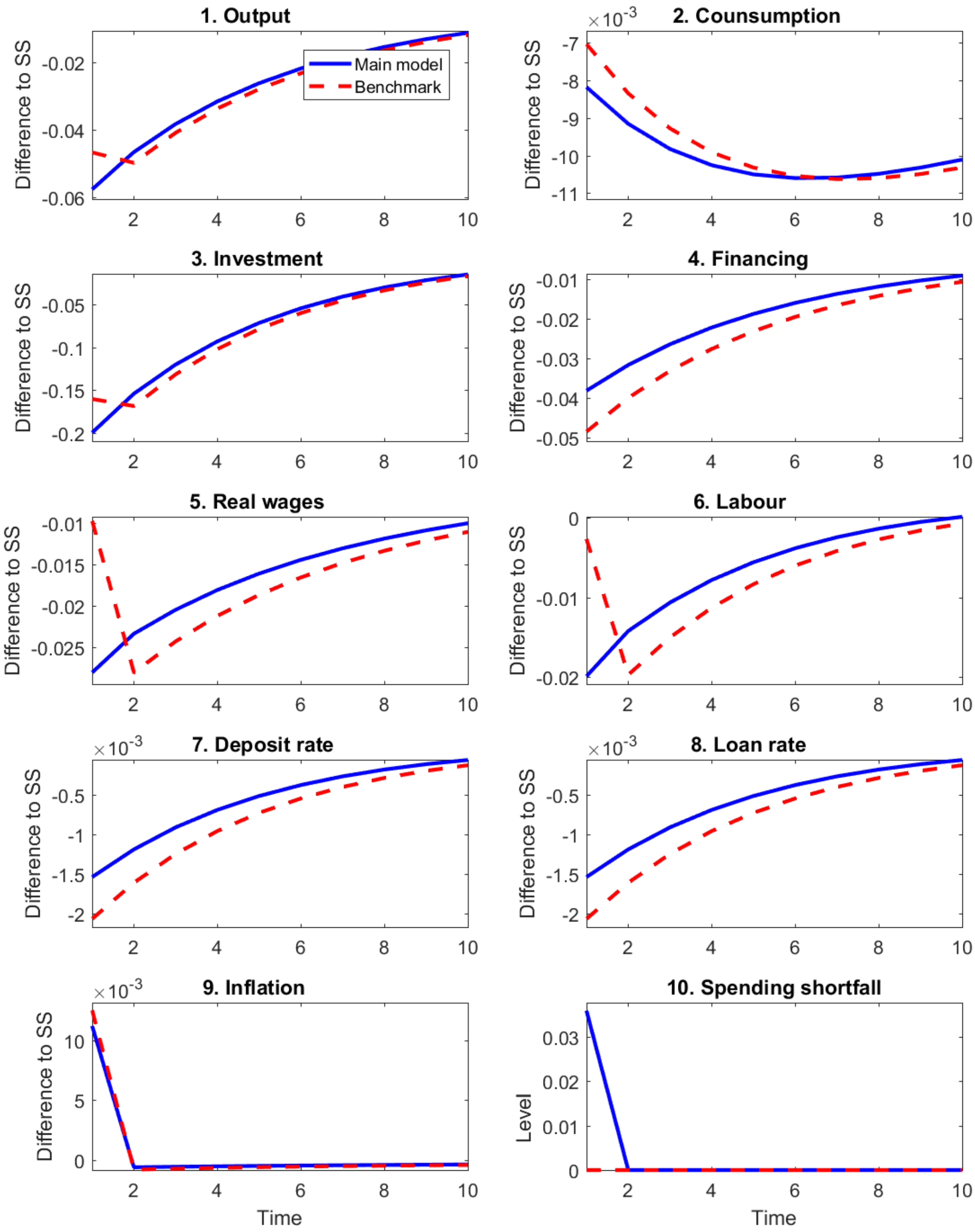

Figure 2.6: Impulse response functions following a large drop in productivity (-4.5× standard deviation). The graph shows deviations from the steady-states for all variables in their logarithmic form but for inflation and the spending shortfall rates that are shown without any transformation. The benchmark model assumes no underspending.

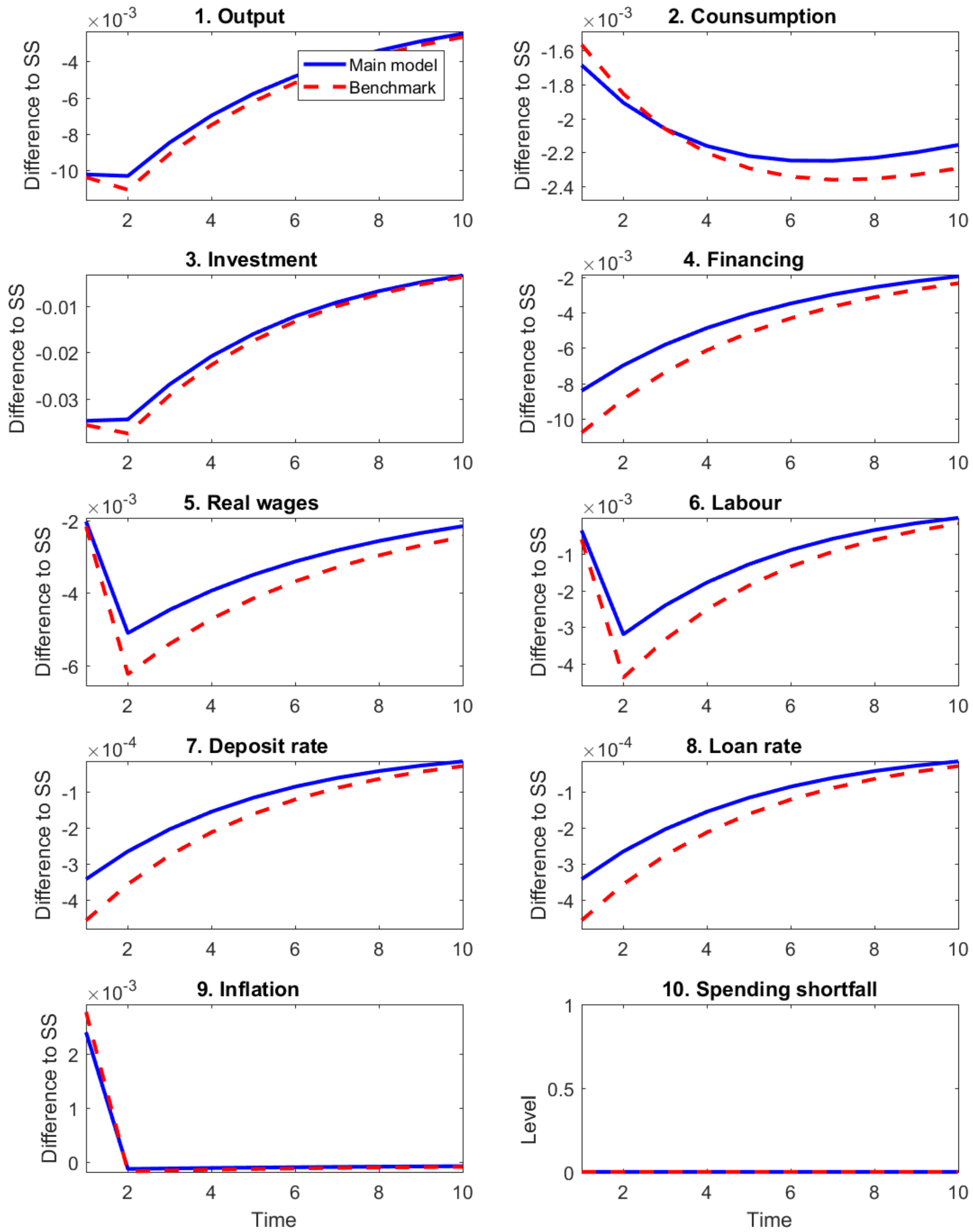

Figure 2.7: Impulse response functions following a moderate positive and negative drop in productivity (-1× standard deviation). The graph shows deviations from the steady-states for all variables in their logarithmic form but for inflation and the spending shortfall rates that are shown without any transformation. The benchmark model assumes no underspending.

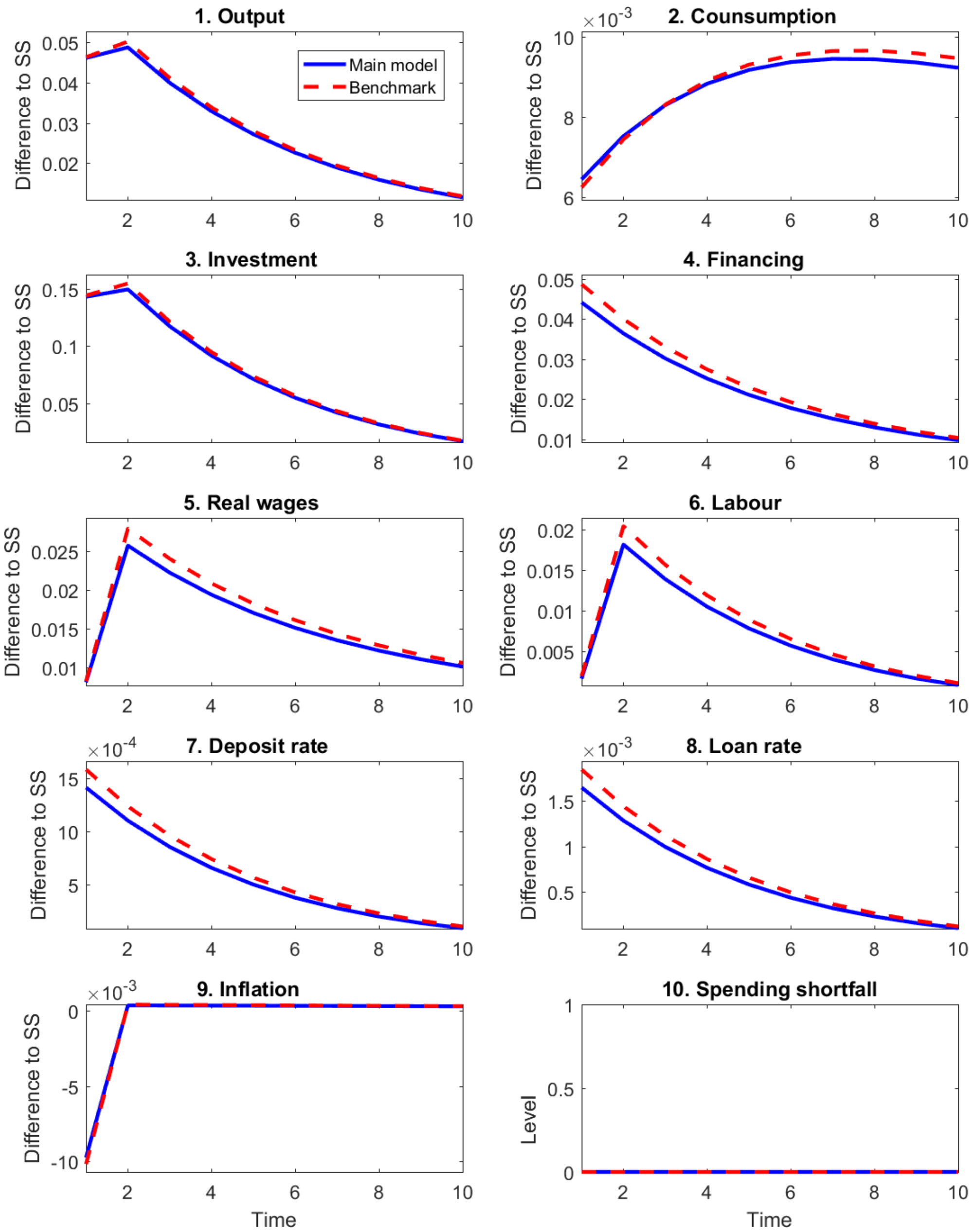

Figure 2.8: Impulse response functions following a large increase in productivity (+4.5× standard deviation). TThe graph shows deviations from the steady-states for all variables in their logarithmic form but for inflation and the spending shortfall rates that are shown without any transformation. The benchmark model assumes no underspending.

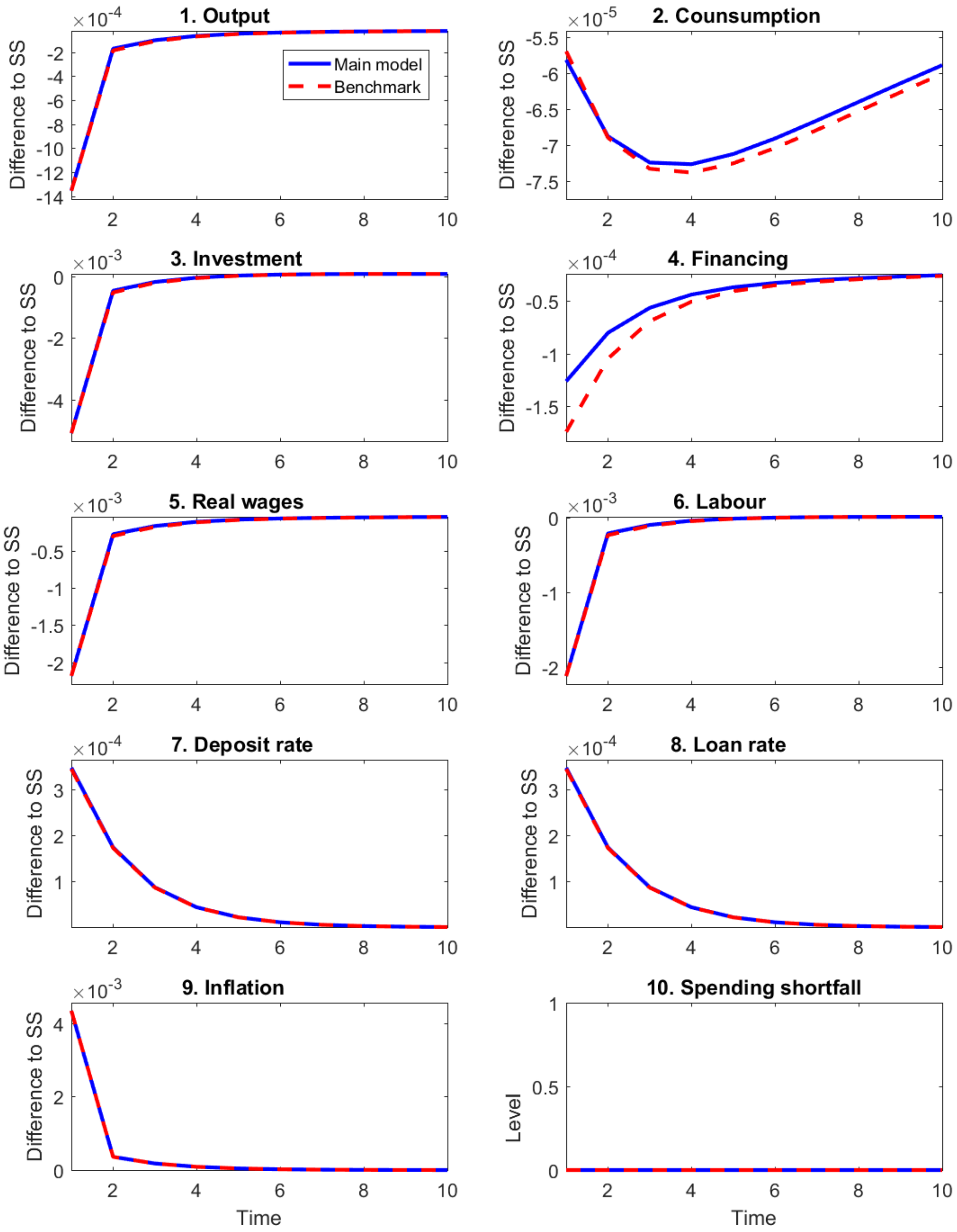

Figure 2.9: Impulse response functions a positive shock ($1\times$ standard deviations) to the process x^H reflecting the increase of money causing pure uinflation effects. The graph shows deviations from the steady-states for all variables in their logarithmic form but for inflation and the spending shortfall rates that are shown without any transformation. The benchmark model assumes no underspending.

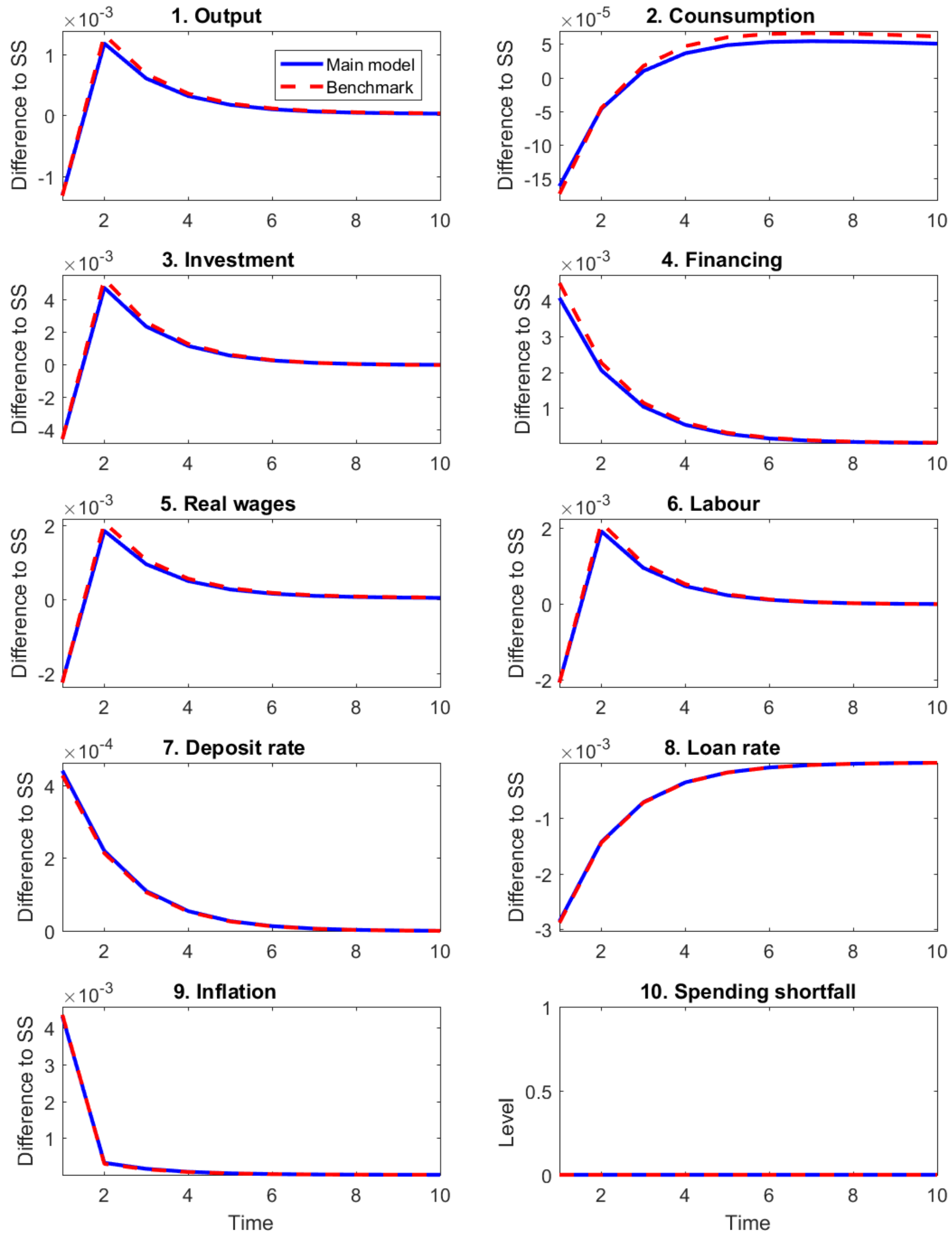

Figure 2.10: Impulse response functions a positive shock (1× standard deviations) to the process x^B reflecting the increase of money through injections in the financial intermediaries. The graph shows deviations from the steady-states for all variables in their logarithmic form but for inflation and the spending shortfall rates that are shown without any transformation. The benchmark model assumes no underspending.

2.A Technical appendix

2.A.1 Model equations

I write the model equations below

$$c_t + k_t - (1-\delta)k_{t-1} = y_t, \tag{2.A.1}$$

$$(1-\alpha)\zeta_t = \omega_t l_t, \tag{2.A.2}$$

$$\alpha\zeta_t = r_t^K k_{t-1}, \tag{2.A.3}$$

$$\frac{1}{c_t} = \beta r_t \mathbf{E}_t \frac{1}{(1+\pi_{t+1})c_{t+1}}, \tag{2.A.4}$$

$$\chi c_t = \omega_t l_t^\eta \tag{2.A.5}$$

$$\xi_{t+1} := \ln\left(\frac{\lambda_{t+1}^\gamma \iota_t^{1-\gamma}}{\gamma(1+\pi_{t+1})^{1-\gamma}}\right). \tag{2.A.6}$$

$$m_t = \frac{1+x_t^B}{1+\pi_t} m_{t-1} \tag{2.A.7}$$

$$r_t^F = \frac{(1+\pi_t)b_t}{(1+\pi_t)b_t + m_{t-1}x_t^B} r_t, \tag{2.A.8}$$

$$\iota_t = b_t + m_{t-1}x_t^B/(1+\pi_t), \tag{2.A.9}$$

$$(m_t^H)^\nu = \psi \frac{r_t}{r_t - 1} c_t, \tag{2.A.10}$$

$$\frac{m_{t-1}}{1+\pi_t} = m_t^H + b_t, \tag{2.A.11}$$

$$\zeta_{t+1} = \min\left\{\frac{\iota_t}{1+\pi_{t+1}}, \left(\gamma\frac{e^{u_{t+1}}}{\lambda_{t+1}^\gamma}\right)^{1/(1-\gamma)}\right\}, \tag{2.A.12}$$

$$y_{t+1} = \min\left\{\frac{e^{u_{t+1}}\iota_t^\gamma}{(1+\pi_{t+1})^\gamma\lambda_{t+1}^\gamma}, \left(\gamma^\gamma\frac{e^{u_{t+1}}}{\lambda_{t+1}^\gamma}\right)^{1/(1-\gamma)}\right\} \tag{2.A.13}$$

$$r_t^F = \mathbf{E}_t\left[\mathbf{1}_{u_{t+1}\le\xi_{t+1}} + \mathbf{1}_{u_{t+1}>\xi_{t+1}} e^{u_{t+1}-\xi_{t+1}}\right]. \tag{2.A.14}$$

2.A.2 Steady-state equilibrium

The steady-state (SS) variables are noted without the time subscript. The SS inflation is given by

$$\pi = x^B + x^H. \tag{2.A.15}$$

The steady-state gross deposit rate r and net capital rental rate are derived from the saving Euler equation

$$r = \frac{1}{\beta}(1+\pi). \tag{2.A.16}$$

$$r^K = \frac{1}{\beta} - 1 + \delta. \tag{2.A.17}$$

I will express the remaining steady-state variables as a function of the critical final productivity shock below which the firm underspends ξ. Then, I will provide a way to solve for ξ so that all steady-state variables are determined.

First note that the cost of financing facing the firms in the steady-state can be expressed as a function of ξ using the financing equation 2.A.14.

$$r^F = e^{-\xi}\Phi\left(-\frac{\xi}{\sigma_u}\right) + \Phi\left(\frac{\xi}{\sigma_u}\right). \tag{2.A.18}$$

Now note that the quantities $\lambda^{\frac{\gamma}{1-\gamma}}y$ and $\lambda^{\frac{\gamma}{1-\gamma}}\zeta$ can be expressed as a function of ξ and the model parameters using steady-state versions of equation and

$$\lambda^{\frac{\gamma}{1-\gamma}}\zeta = \gamma^{\frac{1}{1-\gamma}} \min\left\{1, e^{\xi/(1-\gamma)}\right\}, \tag{2.A.19}$$

$$\lambda^{\frac{\gamma}{1-\gamma}}y = \gamma^{\frac{\gamma}{1-\gamma}} \min\left\{1, e^{\gamma\xi/(1-\gamma)}\right\}. \tag{2.A.20}$$

Given 2.A.19 and 2.A.20, I obtain the ratio y/ζ as a function of the variable ξ and the model's parameters

$$\frac{y}{\zeta} = \frac{1}{\gamma}\max\{1, e^{-\xi}\}. \tag{2.A.21}$$

I combine the labour provision equation 2.2.6 with the good clearing identity ($c + \delta K = y$) and the capital first order condition ($r^K k = \alpha\zeta$) to determine steady-state labour as a function of ξ and the model's parameters

$$l^{-1-\eta} = \frac{\chi}{1-\alpha}\left(\frac{y}{\zeta} - \frac{\alpha\delta}{r^K}\right). \tag{2.A.22}$$

This, combined with the firm's labour first order condition ($w = (1-\alpha)\zeta/l$), determines the steady-state production cost λ as a function of ξ, ζ and the model's parameters

$$\lambda = \left(\frac{r^K}{\alpha}\right)^{\alpha}\left(\frac{\zeta}{l}\right)^{1-\alpha}. \tag{2.A.23}$$

Combine 2.A.22 and 2.A.23 with 2.A.19 to get ζ as a function of ξ and the model's parameters. The later result yields the steady-state production cost λ, the steady-state output y, the steady-state wage $w = (1-\alpha)\zeta/l$, the steady-state capital $k = \alpha\zeta/r^K$, the steady-state consumption $c = y - \delta\alpha\zeta/r^K$ and the steady-state real money $(m^H)^{\nu} = \psi\frac{r}{r-1}c$ as a function of ξ and the model's parameters. The level of loans issued by banks is deduced as a function of wages and ξ using the definition 2.A.6

$$\iota = (1+\pi)\left(\frac{\gamma e^{\xi}}{\lambda^{\gamma}}\right)^{\frac{1}{1-\gamma}}. \tag{2.A.24}$$

The steady-state real money is derived from the money demand condition 2.A.10. The steady-state real saving is deduced by combining the money clearing equation 2.A.11 with equation 2.A.9

$$b = \frac{\iota - x^B m^H}{1+x^B}, \tag{2.A.25}$$

and the total stock of real money is $m = (1+x^B)(m^H + b)$.

After expressing all the steady-state variable of the model as a function of ξ and the model parameters, I solve for ξ using the bank zero profit condition

$$b = \frac{r^F}{r}\iota. \tag{2.A.26}$$

For the model's parameters I consider here, I verify numerically that the latter equation has a unique solution. Once, the steady-state variable ξ is known, the remaining steady-state variables are determined in a straightforward manner.

The sign of the the variable ξ determines whether there is underspending in the steady-state.

- **Case $r^F \geq 1$:** Assuming $r^F \geq 1$ implies $\xi \leq 0$. This implies that there is no underspending in the steady-state.

- **Case $r^F < 1$:** Assuming $r^F < 1$ implies $\xi < 0$. This implies that the firm underspends in the steady-state.

Chapter 3

Underspending in a Network of Industries Linked Through Input-Output Relationships

3.1 Introduction

The previous chapter introduced a mechanism whereby firms may prefer to distribute cash to shareholders after unexpected drops in productivity rather than investing in the production process. This chapter considers the same mechanism in the context of a network of industries linked through input-output relationships. I show that if productivity drops in parts of the productive sector, this can propagate through the production cost channel and cause underspending in industries where productivity matches or beats expectations (downstream transmission). In addition, underspending can affect the suppliers of industries witnessing drops in productivity, thus propagating underspending to industries where productivity does not deteriorate (upstream transmission). The nature of the input-output network linking firms influences the way underspending can propagate from underperforming industries to the rest of the productive sector.

I build on the framework in Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) that I modify by separating the firm's financing decision from its spending decision and by introducing money and capital accumulation in a similar way to the model presented in chapter 2. I provide several theoretical results characterising the level of underspending in the economy and its impact on other model variables. These results help simulate the model while dealing with the issue of occasionally binding spending constraints. I then present industry aggregation results where the characteristics of the input-output network are linked to the likelihood of firms' underspending as the number of industries in the economy grows large. To this effect, I focus on two particular networks: the symmetric fully connected network, where every firm equally uses all industries as a provider of intermediary input, and the star network, where a single industry is the only provider of intermediary input to other industries. I find that in the case of the star network, the model's mechanism is maintained for highly disaggregated economies as the probability for all firms to underspend remains stable independently of the number of industries in the economy. The behaviour of underspending is different when symmetric, fully connected networks link industries. In the latter case, the probability of all firms underspending converges to zero for economies with a large number of industries.

When considering the underspending mechanism in a setup with multiple industries, underspending can, a priori, happen in any subset of industries. This complex "occasionally binding constraints" feature significantly complicates the simulation of the model. I provide several theoretical results that enable the numerical simulation of the model while accounting for all different possibilities in terms of the subset of industries where spending is not constrained. To this effect, I develop an ad hoc simulation method that uses perturbation techniques to deal with the forward-looking model equations while solving the other model equations without recourse to perturbation techniques. This numerical method uses ideas similar to those used to develop the "Exact Today" method presented in Den Haan, Kobielarz, and Rendahl (2016). However, this chapter's ad hoc simulation method is substantially simpler to implement than the "Exact Today" method. It is also better suited to the model presented in this chapter.

Simulation results are shown for a simple economy composed of two industries following an unexpected negative shock to one of the two industries. The model of this chapter is compared to a benchmark model where the previous financing always constrains spending. Through the underspending mechanism, lower firms' spending decreases labour compensation, thus causing labour supply to drop. Lower labour supply causes lower output and therefore, lower investments as consumers tend to smooth consumption variations. The labour supply channel is key to the functioning of the underspending mechanism. To further demonstrate the importance of the labour channel, I provide simulation results for an alternative calibration assuming a higher elasticity of labour supply. The higher labour elasticity assumption causes firms' underspending to have more significant effects on aggregate variables.

Even when productivity remains stable in parts of the productive sector, a large enough negative surprise shock in a subset of industries can propagate to the whole productive sector. Besides the usual downstream propagation through industrial cost functions that depend on the productivities of suppliers, propagation happens through two main demand channels. First of all, a negative shock in parts of the productive sector causes a drop in households' demand, thus affecting demand for the goods produced by the industries where productivity remains stable or improves. Moreover, the parts of the productive sector where the shock operates reduce demand for intermediate goods produced by the remaining industries. These effects can combine to reduce the marginal return on spending in stable and improving industries, pushing their spending levels below the financing constraint.

In the absence of the underspending mechanism, production is more stable in the industries that do not witness a productivity shock despite lower aggregate output and consumption. This stability is due to a substitution effect where production is directed, through price change effects, from the less productive industries where the shock originated to industries where productivity remains stable or improves. However, this substitution effect is moderated by the underspending mechanism. The way underspending affects stable industries is clearer when simulating the model assuming an intermediate production sector composed of two industries linked through a star input-output network. This is an economy with a single industry producing an intermediate input used by all intermediate producers (source industry) and another industry where firms use the input produced by the source industry to produce a good that is dedicated to final production (sink industry). A large enough negative productivity shock in the source industry causes underspending in both industries. Firms' underspending drops output further down in both industries as both spend less on wages and attract less of the elastic labour supply. However, lower output in the source industry has an indirect effect on the sink industry. The price of the scarcer good produced by the source industry increases, thus increasing the production cost of the sink industry. Underspending affects the sink industry in two ways: by reducing its labour demand and by raising its production cost. Overall, the sink industry is more impacted by underspending than the source industry.

The model presented in this chapter considers the effect of financial frictions in an economy with multiple industries. This friction is linked to a working capital constraint similar to the one presented in Christiano and Eichenbaum (1992). The previously raised financing (or working capital) represents an upper limit to current spending but only if spending remains attractive enough to the firm. Following unexpected drops in productivity, increases in production costs or drops in demand, the firm might find it suboptimal to produce at a level of spending that corresponds to the financing constraint set before the unpredicted shock hits its profits. In the context of the model presented here, the mechanism only functions following unexpected adverse shocks. The underspending mechanism has an asymmetric impact on the firms' behaviour which can help justify the asymmetry in the evolution of aggregate variables over the business cycle (documented in Neftci (1984), among others). Given the importance of sectoral shocks' aggregation to the functioning of the underspending mechanism, I build on the framework in Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012). In this paper, the authors argue that even in a very disaggregated economy, industry-level independent productivity shocks can lead to non-negligible fluctuations of aggregate variables.[1] This is the case when the input-output network linking industries is such as a small number of industries play a disproportionately important role as input supplies to others. The model presented here builds on this framework and extends it in a direction where some of the asymmetries of aggregate variables can find their roots in the sectoral behaviour of firms.

[1] A very disaggregated economy means an economy with a large number of industries subject to independent idiosyncratic productivity shocks.

The studied underspending mechanism relies on working capital frictions. It is therefore related to the question of macroeconomic response to sectoral distortions as studied in Bigio and LaO (2020). In the latter paper, the authors show that input-output networks can amplify the effects of sectoral level inefficiencies. Focusing on financial inefficiencies, they show that the U.S. input-output structure amplified distortions linked to the excess bond premia (as defined in Gilchrist and Zakrajek (2012)) by roughly a factor of two during the 2008-2009 crisis. While the focus in this chapter is on when the financing constraints cease to be relevant to the firms' spending behaviour, the wedge introduced by the cost of financing affects the steady-state in a way that is consistent with the theoretical results presented in Bigio and LaO (2020). Financing constraints increase the production costs from the firm's perspective by introducing a wedge between real inputs prices and their marginal product. In the same vein as Bigio and LaO (2020), Baqaee and Farhi (2019) provide theory of aggregation in inefficient economies. The authors generalise Hulten Theorem (Hulten (1978)) to equilibrium allocations away from the efficient one. Baqaee and Farhi (2019) assume that both labour supply and distortion wedges are exogenous, while Bigio and LaO (2020) assume endogenous labour supply and exogenous distortion wedges. The model presented here assumes that production input includes capital besides labour and intermediary inputs and that supply for all these inputs is endogenous. Moreover, the cost of financing (the only distortion wedge) is endogenous and results from the optimising behaviour of borrowers and lenders. Finally, an additional layer of complexity is introduced through the explicit modelling of money demand as a combination of Households' demand emanating from money in the utility function and working capital demand from producers.[2]

The remainder of this chapter is organised as follows. Section 3.2 presents the assumptions of the model and some of the consequences of the agents' optimising behaviour. Section 3.3 provides more theoretical results regarding the underspending mechanism. The network effects on the model's main mechanism are presented in section 3.4. This section also provides some aggregation results in the case of two notable input-output network structures. The simulation

[2]Other related works focusing misallocation in economies with input-output relations include Jones (2011b) and Liu (2019), the reader can refer to Bigio and LaO (2020) for a more comprehensive literature review.

method and the model calibration are described in section 3.5. In addition, section 3.5 presents and comments the simulation results. Section 3.6 concludes.

3.2 The model

In this section, I present a general equilibrium model where firms are linked through input-output relationships and can enter an underspending mode following unexpected drops in productivity. In this setup, n intermediate goods are produced by industries indexed $i = 1, ..., n$. These industries are constrained by a Cobb-Douglas production function, and they use each other's output as intermediary input. The use by each industry of inputs provided by other industries is given by the use input-output matrix $W := (w_{ij})_{i,j=1...n}$ where $\sum_{j=1}^{n} w_{ij} = 1$. The mechanism presented in chapter 2 is adapted to allow for firms to be connected through input-output relationships. The subsection below provides the details underlying the model extension.

3.2.1 Households

Households derive utility from the consumption of a single good c_t, leisure $1-l_t$ and real money holdings m_t^H

$$\mathscr{U}(c_t, 1-l_t, m_t^H). \tag{3.2.1}$$

$\mathscr{U}$ verifies the first and the second derivatives conditions: $\mathscr{U}_c > 0$, $\mathscr{U}_{m^H} > 0$, $\mathscr{U}_{1-l} > 0$, $\mathscr{U}_{c,c} < 0$, $\mathscr{U}_{1-l,1-l} < 0$ and $\mathscr{U}_{m^H,m^H} < 0$. Households decide their consumption c_t, labour supply l_t, capital k_t, deposits b_t and real money holdings m_t^H by maximizing their lifelong expected utility

$$\max_{c_s, l_s, m_s^H, b_s, k_s} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \mathscr{U}(c_s, 1-l_s, m_s^H), \tag{3.2.2}$$

While being subject to the sequence budget constraint

$$b_s + p_s m_s^H + p_s k_s + p_s c_s = \omega_s l_s + r_{s-1} \frac{b_{s-1}}{1+x_{s-1}^M} + \frac{p_{s-1} m_{s-1}^H}{1+x_{s-1}^M} + (1-\delta+r_s^K) p_s k_{s-1} + \mathscr{T}_s, \tag{3.2.3}$$

where p_t is the price of the consumption good, ω_t the nominal wage, r_t the (gross) deposit rate, r_t^K the (net) capital rent and $\mathscr{T}_t$ is the lump-sum transfers from the households' owned firms and banks. In line with the monetised real business cycle literature, all the nominal variables (including b_t, ω_t, p_t and $\mathscr{T}_t$) are expressed as fractions of the previous stock of money that grows at a rate x_t^M.

Deposit rates r_t and capital rents r_t^K must satisfy the Euler intertemporal equations

$$\mathscr{U}_{c,t} = \frac{\beta r_t}{1 + x_t^M} \mathbb{E}_t \frac{p_t}{p_{t+1}} \mathscr{U}_{c,t+1}, \tag{3.2.4}$$

$$1 = \beta \mathbf{E}_t \frac{\mathscr{U}_{c,t+1}}{\mathscr{U}_{c,t}} (1 - \delta + r_t^K). \tag{3.2.5}$$

Households provide labour as per the condition

$$\mathscr{U}_{1-l,t} = \frac{\omega_t}{p_t} \mathscr{U}_{c,t}. \tag{3.2.6}$$

Finally, the representative household sets its money holding to satisfy

$$\mathscr{U}_{m^H,t} = \mathscr{U}_{c,t} \frac{r_t - 1}{r_t}. \tag{3.2.7}$$

From the condition above and the derivative conditions satisfied by the utility function $\mathscr{U}$, one can see that it is more costly for households to hold positive money balances when the nominal deposit rate r_t is high and when consumption c_t is low.

3.2.2 Financial intermediation and monetary policy

As in chapter 2, I follow Christiano and Eichenbaum (1992) and assume that a direct monetary channel is operated by injecting money in financial intermediaries to help finance the loans extended to production firms. The representative bank provides the firms with the aggregate loan ι_t (expressed as a fraction of the previous money stock) charging the gross nominal rate r_t^F. The bank finances its loans operations using households deposits b_t and a monetary injection from the central bank that is proportional to the existing nominal stock of money

$$\iota_t = b_t + x_t^M, \tag{3.2.8}$$

where x_t^M is the relative change in the stock of money. The bank profit (expressed as a fraction of the previous money stock) is given by

$$\Pi_{t+1}^B = r_t^F \iota_t - r_t b_t. \tag{3.2.9}$$

The monetary authority provides the financial intermediaries with the new money in the goal of easing credit conditions for firms. This prevents the increase in the money supply from being distributed equally to all agents and guarantees a role for money in the model's fluctuations. Following Christiano and Eichenbaum (1995), I assume that the bank's profit is zero ($\Pi_{t+1}^B = 0$). This yields

$$r_t^F \iota_t = r_t b_t. \tag{3.2.10}$$

by controlling the money injection x_t^M, the central bank affects the lending rate

$$r_t^F = \frac{1}{1 + x_t^M / b_t} r_t. \tag{3.2.11}$$

I assume that the money growth x_t^M follows the AR(1) process

$$x_t^M = (1 - \rho_M) x^M + \rho_M x_{t-1}^M + \sigma_M v_t^M, \tag{3.2.12}$$

where x^M is the steady-state value of x_t^M, σ_M is the volatility of innovations, ρ_M is the autocorrelation parameter and v_t^M is an *i.i.d* error term that is independent of all other shocks in the model.

The assumption that money is directly injected into the financial intermediaries can be justified by the ability of central banks to affect lending through the use of open-market operations (Christiano and Eichenbaum (1992)). All the stock of money available at the end of the production cycle is inherited by the households and is either kept in real cash balances or deposited with the financial intermediary

$$p_t m_t^H + b_t = 1. \tag{3.2.13}$$

The latter money clearing condition, combined with the household demand for money emanating from utility maximisation as described by equation 3.2.7, determines the price of the consumption good p_t in the model.

3.2.3 Final good producer

I assume the existence of a final good producer that transforms the output of the industries $1, ..., n$ into a final good y_t that is then used by households to consume or to accumulate capital: $y_t = k_t - (1-\delta)k_{t-1} + c_t$. The final production technology is

$$y_t = \prod_{i=1}^{n} \left(\frac{y_{i,t}}{\theta_i} \right)^{\theta_i}, \tag{3.2.14}$$

where $\theta_i \geq 0$ is the share of good i in the final production and $\sum_{j=1}^{n} \theta_i = 1$. The representative final good producer maximises profit to decide the basket of used intermediate inputs $(y_{1,t}, ..., y_{n,t})$

$$\max_{y_{1,t},\cdots,y_{n,t}} p_t y_t - \sum_{j=1}^{n} p_{i,t} y_{i,t}. \tag{3.2.15}$$

Through first order conditions

$$p_{i,t} y_{i,t} = \theta_i p_t y_t \tag{3.2.16}$$

This and the final production technology lead to the below final good price expression

$$p_t = \prod_{i=1}^{n} p_{i,t}^{\theta_i}. \tag{3.2.17}$$

3.2.4 Firms

The intermediate goods are produced by industries indexed $i = 1, ..., n$ that are constrained by a Cobb-Douglas production function and that use each other's output as intermediary input

$$x_{i,t} := f_i(z_{i,t}, k_{i,t}, l_{i,t}, X_{i,t}) = z_{i,t} k_{i,t}^{\alpha_K} l_{i,t}^{\alpha_L} \prod_{j=1}^{n} x_{i,j,t}^{\alpha_I w_{i,j}}, \tag{3.2.18}$$

where $X_{i,t} := (x_{i,1,t}, ..., x_{i,n,t})'$ denotes the vector of the outputs $x_{i,j,t}$ of industries $j = 1, .., n$ used as intermediary inputs by industry i, $l_{i,t}$ is the labour employed by industry i, $k_{i,t}$ is the capital rented by the same industry and $z_{i,t}$ is an industry-specific total productivity factor. The use by each industry of intermediary inputs provided by other industries is determined by the use input-output matrix $W = (w_{ij})_{i,j=1...n}$ where $\sum_{j=1}^{n} w_{ij} = 1$ for $i = 1, ..., n$ and the overall use of intermediary inputs is governed by the elasticity parameter α_I. The share of capital (respectively labour) is noted α_K (respectively α_L) and I assume decreasing return to scale ($\alpha_K + \alpha_L + \alpha_I < 1$). Firms belonging to the same industry i are assumed to be identical, with log-normal productivity z_i

$$z_{i,t} = \exp(u_{i,t}), \tag{3.2.19}$$

where $u_{i,t}$ $i = 1, ..., n$ are industry-specific shocks that are assumed to be $AR(1)$

$$u_{i,t} = \rho_i u_{i,t-1} + \sigma_i e_{i,t},$$

σ_i is an industry-specific volatility parameter, and ρ_i denotes an industry-specific mean-reversion parameter. The productivity processes are therefore driven by the *i.i.d.* normal innovations e_i that are also assumed to be independent across industries.

Firms finance production using banks' issued loans carrying the gross interest rate r_t^F. At time t, the firms decide the loan amount $\iota_{i,t}$ that will potentially be spent in the next period's production process. To do so, the firm maximises its expected profit that is a function of the expected sales proceed $p_{i,t+1}f_i(.)$, the level of financing ι_i and the cost of rising debt r_t^F

$$\max_{\iota_i(t)} \mathbb{E}_t p_{i,t+1} f_i(z_{i,t+1}, k_{i,t+1}, l_{i,t+1}, X_{i,t+1}) - r_t^F \frac{\iota_{i,t}}{1 + x_t^M}, \tag{3.2.20}$$

subject to the future financial resource constraint

$$p_{t+1} r_{t+1}^K k_{i,t+1} + \omega_{t+1} l_{i,t+1} + \sum_{j=1}^{n} p_{j,t+1} x_{i,j,t+1} \leq \frac{\iota_{i,t}}{1 + x_t^M}. \tag{3.2.21}$$

Note that, as is the case of other nominal model variables, ι_t is expressed as a fraction of the previous period money stock. The constraint on spending in the next period needs to be corrected to take into account the growth of the money stock x_t^M.

The main mechanism of the model functions in a similar way to the single industry case. A firm belonging to an industry i decides its financing level $\iota_{i,t}$ at time t, raises loan financing before discovering the new productivity $e^{u_{i,t+1}}$, new sale prices $p_{i,t+1}$ and the new production costs it faces. At time $t+1$, the firm assesses its own productivity as well as the prevailing wages and prices then decides its overall spending $\zeta_{i,t+1} := r_{t+1}^K p_{t+1} k_{i,t+1} + \omega_{t+1} l_{i,t+1} + \sum_{j=1}^{n} p_{j,t+1} x_{i,j,t+1}$ in order to maximise its profit subject to the previously determined spending constraint ι_i

$$\max_{x_{i,1,t+1},\ldots,x_{i,n,t+1};l_{i,t+1},k_{i,t+1}} p_{i,t+1} f_i(z_{i,t+1}, k_{i,t+1}, l_{i,t+1}, X_{i,t+1}) - r_{t+1}^K p_{t+1} k_{i,t+1} - \omega_{t+1} l_{i,t+1} - \sum_{j=1}^{n} p_{j,t+1} x_{i,j,t+1}, \tag{3.2.22}$$

$$\text{s.t. } r_{t+1}^K p_{t+1} k_{i,t+1} + \omega_{t+1} l_{i,t+1} + \sum_{j=1}^{n} p_{j,t+1} x_{i,j,t+1} \leq \frac{\iota_{i,t}}{1 + x_t^M}. \tag{3.2.23}$$

The first order conditions of the spending problem above write

$$p_{j,t+1} x_{i,j,t+1} = \frac{\alpha_I}{\alpha} w_{i,j} \zeta_{i,t+1}, \tag{3.2.24}$$

$$p_{t+1} r_{t+1}^K k_{i,t+1} = \frac{\alpha_K}{\alpha} \zeta_{i,t+1}, \tag{3.2.25}$$

$$\omega_{t+1} l_{i,t+1} = \frac{\alpha_L}{\alpha} \zeta_{i,t+1}, \tag{3.2.26}$$

where $\alpha := \alpha_K + \alpha_L + \alpha_I$ denotes the return to scale of intermediate production. Combining these first order conditions with the production technology leads the following expression for the output of industry i

$$x_{i,t+1} = z_{i,t+1} \frac{\zeta_{i,t+1}^{\alpha}}{\kappa_{i,t+1}}, \tag{3.2.27}$$

where κ_i is an index representing the cost of production facing firms within industry i. the index κ_i is a function of the rental rate of capital r_{t+1}^K, the good prices $P_{t+1} := (p_{1,t+1}, ..., p_{n,t+1})'$ and wages ω_{t+1}

$$\kappa_{i,t+1} := \frac{\alpha^{\alpha}}{\alpha_K^{\alpha_K} \alpha_L^{\alpha_L} \alpha_I^{\alpha_I}} \left(p_{t+1} r_{t+1}^K\right)^{\alpha_K} \omega_{t+1}^{\alpha_L} \prod_{j=1}^{n} \left(\frac{p_{j,t+1}}{w_{i,j}}\right)^{\alpha_I w_{i,j}}, \tag{3.2.28}$$

where, by convention, $\left(\frac{p_{j,t+1}}{w_{i,j}}\right)^{\alpha_I w_{i,j}} = 1$ when $w_{i,j} = 0$. One can therefore write the production problem at time $t+1$ as a mere spending problem

$$\max_{\zeta_{i,t+1}} p_{i,t+1} z_{i,t+1} \frac{\zeta_{i,t+1}^{\alpha}}{\kappa_{i,t+1}} - \zeta_{i,t+1}, \tag{3.2.29}$$

$$\text{s.t. } \zeta_{i,t+1} \leq \frac{\iota_{i,t}}{1 + x_t^M}. \tag{3.2.30}$$

The firm chooses whether to spend all the raised financing $\zeta_{i,t+1} = \iota_{i,t}/(1 + x_t^M)$ (constrained case) or to spend less than the raised financing level $\zeta_{i,t+1} < \iota_{i,t}/(1 + x_t^M)$ (unconstrained case) as per the following equation

$$\zeta_{i,t+1}^{1-\alpha-\gamma} = \min\left\{\left(\frac{\iota_{i,t}}{1 + x_t^M}\right)^{1-\alpha-\gamma}, \alpha e^{u_{i,t+1}} \frac{p_{i,t+1}}{\kappa_i(P_{t+1})}\right\}. \tag{3.2.31}$$

This equation is derived from the fact that when the firm is unconstrained, the marginal return on spending matches the marginal return on returning cash to shareholders without spending on the production process (i.e. one)

$$\frac{\partial p_{i,t} x_{i,t+1}}{\partial \zeta_{i,t+1}} = 1. \tag{3.2.32}$$

The industry i is unconstrained if, at the spending cnstraint $\frac{\iota_{i,t}}{1+x_t^M}$, the marginal return on spending is less than one

$$\frac{\partial p_{i,t}x_{i,t+1}}{\partial \zeta_{i,t+1}} = \alpha e^{u_{i,t+1}}\frac{p_{i,t+1}}{\kappa_{i,t+1}}\frac{\iota_{i,t}^{\alpha-1}}{(1+x_t^M)^{\alpha-1}} \leq 1. \tag{3.2.33}$$

The intuitions from the single-industry model are maintained, with firms entering unconstrained spending when their industry's log productivity is lower than a critical value determined by the previous financing, money growth x^M, sale prices p_i and production costs κ_i. The input-output relationships influence the model's main mechanism through the production cost channel by impacting the production cost indices $\kappa_{i,t+1}$ and through the demand channels affecting the sale prices $p_{i,t+1}$. A negative shock originating in a subset of industries can propagate to other industries by increasing their production costs, thus making spending unconstrained even in industries with improving productivities. Additionally, the negative shock can curtail demand in the affected industries, thus affecting these industries' suppliers. The effect the production network has on the underspending mechanism is discussed in more detail in section 3.4.

At this stage, one can derive an industry-specific equation linking the level of financing within industry i to the expected productivity within the same industry, expected sale prices and expected production costs. From the financing problem described by 3.2.20 and 3.2.21

$$\mathbb{E}_t\left[\mathbf{1}_{\{\iota_{i,t}^{1-\alpha}<\alpha(1+x_t^M)^{1-\alpha}e^{u_{i,t+1}}\frac{p_{i,t+1}}{\kappa_{i,t+1}}\}}\alpha e^{u_{i,t+1}}\frac{p_{i,t+1}}{\kappa_{i,t+1}}\right] = r_t^F\left(\frac{\iota_{i,t}}{1+x_t^M}\right)^{1-\alpha}. \tag{3.2.34}$$

Note that the marginal effect of financing ι_t on expected firms' profit is only positive if the future spending is constrained; otherwise, the marginal effect of financing is nil. The financing equation 3.2.34 confirms the importance of the decreasing returns to scale assumption in guaranteeing finite levels of financing demands. Additionally, under the decreasing returns to scale assumption, financing levels increase with expected sale prices and productivity while being inversely related to financing costs and the expected production costs as summarised by κ_i. The production cost index κ_i is key in propagating shocks originating in a single industry to the rest of the production network: a negative shock impacting industry j reduces good j supply and as a result increases the price of the same good. This in turn increases the production cost

κ_i facing all industries i where the good j is used (these are the industries i such as $w_{i,j} > 0$).

3.2.5 Market clearing conditions

For clarity, I provide the clearing conditions corresponding to all markets below.

- The markets for intermediate goods $i = 1, ..., n$ where the supply matches the demand emanating from intermediate and the final production

$$y_{i,t} + \sum_{j=1}^{n} x_{j,i,t} = x_{i,t}, \tag{3.2.35}$$

where $x_{i,t}$ is the overall production of good i at time t

$$x_{i,t} = \frac{\zeta_{i,t}^{\alpha}}{\kappa_{i,t}} e^{u_{i,t}}. \tag{3.2.36}$$

- The loan market where the combination of households' supply and new money meets the production sector demand

$$b_t + x_t^M = \sum_{i=1}^{n} \iota_i(t). \tag{3.2.37}$$

- The market for physical capital where the supply by household matches the demand from all industries

$$k_{t-1} = \sum_{i=1}^{n} k_{i,t}. \tag{3.2.38}$$

- The market for labour, where the demand from producers matches households' supply

$$\sum_{i=1}^{n} l_{i,t} = l_t. \tag{3.2.39}$$

3.3 Model theoretical results

I present below several theoretical results that provide insights about the functioning of the model's underspending mechanisms and aid in designing methods for its numerical simulation.

As it will be shown below, many model variables can be expressed as a function of nominal output, nominal spending, exogenous productivity shocks and previously determined state variables. This model structure proves crucial in simplifying the simulation procedure. To simplify the exposition of the model results, I assume that if there are m industries where the financing constraints are not binding, then these industries have indices $1, ..., m$. This notational assumption can be made without any loss of generality and will be adopted for the remainder of this section.[3] Most of the results of this section are also general enough to accommodate various assumptions regarding the households' utility function. A particular utility function will be specified when needed to obtain an explicit households' labour supply function.

The following lemma is derived from the goods' clearing equations and the model's first-order conditions and shows that firm revenues are only a function of current firms' expenditure and the contemporaneous nominal output. The proof of this lemma and the proofs of the other results presented in this section are detailed in appendix 3.A.1.

Lemma 1. *The firms revenues are determined by the current levels of firms' expenditure and current nominal output*

$$REV_t = \frac{\alpha_I}{\alpha} W' Z_t + y_t^N \Theta \tag{3.3.1}$$

where $REV_t := (p_{1,t}x_{1,t}, ..., p_{n,t}x_{n,t})'$ is the vector of industry revenues expressed as a fraction of the previous period's money stock, $Z_t := (\zeta_{1,t}, ..., \zeta_{n,t})'$ is the industries' expenditures vector also expressed as fractions of the previous period's stock of money, $\Theta := (\theta_1, ..., \theta_n)'$, W the input-output matrix and $y_t^N := p_t y_t$ the nominal output (expressed as a fraction of the previous money stock).

Proof. The proof is presented in appendix 3.A.1. ■

In the case where all industries are constrained by their previous period financing, their expenditure is predetermined, and so is their nominal demand of various intermediary inputs. This means that, when all financing constraints are holding, nominal output plays a central

[3] The notational assumption would imply that industry notations might change from one time period t to the other. We will be focusing on solving the model one time step at a time, so this notational choice should cause no confusion.

role in this model as the only contemporaneous variable determining overall nominal demand for all goods produced by the economy. The following proposition extends the centrality of nominal output to the case where some industries' spending is not constrained, as it expresses the expenditure of unconstrained industries as a function of the current nominal output and predetermined model state variables.

Proposition 1. *The firms' expenditure are function of the firm financing raised in the previous period and nominal output. Either the level of expenditure is equal to the previously raised financing (binding constraint industries) or, for industries where the constraint is not binding, it is determined by current nominal output and previous period financing.*

$$Z_{1:m,t} = \mathscr{L}_m \left\{ \frac{\alpha_I}{1 + x^M_{t-1}} W'_{1:m,m+1:n} I_{m+1:n,t-1} + \alpha y^N_t \Theta_{1:m} \right\}, \tag{3.3.2}$$

where $\mathscr{L}_m := (\mathbb{I}_{m,m} - \alpha_I W'_{1:m,1:m})^{-1}$ *is the Leontief matrix corresponding to the subset of unconstrained industries* $i = 1, ..., m$, $\mathbb{I}_{m,m}$ *denotes the identity matrix of size* $m \times m$ *and if* $M := (m_{i,j})$ *is a matrix, then* $M_{p:q,r:s}$ *denotes the sub-matrix* $M_{p:q,r:s} := (m_{i,j})_{i \in \{p,\dots,q\}, j \in \{r,\dots,s\}}$.

Proof. The proof is presented in appendix 3.A.1. ■

Combining proposition 1 and lemma 1 shows that revenues $rev_{i,t} := p_{i,t} x_{i,t}$ are also determined by nominal output and previous period financing. Using the first-order equations and market clearing conditions, it follows from proposition 1, that other nominal variables are also determined by the current nominal output and previous period investment levels. In other words, the full current shocks impact on several nominal variables is reflected through the current nominal output. These include firm revenues, firms' spending levels, industry costs of labour and capital, the overall wages paid and the overall rent cost of capital. Other variables depend directly on specific industry shocks. These include the prices $p_{i,t}$ and the financing levels $\iota_{i,t}$. The prices $p_{i,t}$ adjust to reflect the varying levels of production across industries, while the financing levels are forward-looking variables that depend on the agents' expectations of the next period realization of industry-specific shocks.

Proposition 2. *The revenues $rev_{i,t} := p_{i,t}x_{i,t}$, the costs of intermediary inputs $p_{i,t}x_{i,j,t}$, the costs of labour $\omega_t l_{i,t}$ and the costs of capital $p_t r_t^K k_{i,t}$ are determined by the current nominal output and a predetermined state variables, namely the previously determined industry financing.*

Proof. The proof is presented in appendix 3.A.1. ■

As explained above, prices depend both on nominal output (through revenues and firms' expenditures) and on industry-specific shocks as per the proposition below.

Proposition 3. *The matrix $\mathbb{I}_{n,n} - \alpha W'$ is invertible and prices are a function of the level of firms' expenditures, firms' revenues and industry shocks as per the equation below*

$$\tilde{P}_t = \mathscr{L}' \left\{ A + \left[(\alpha_K + \alpha_L)\tilde{\zeta}_t - \alpha_K \tilde{k}_{t-1} - \alpha_L \tilde{l}_t \right] \mathbf{1}_{n,1} + R\tilde{E}V_t - \alpha \tilde{Z}_t - U_t \right\}, \tag{3.3.3}$$

where $\mathscr{L}_t = (\mathbb{I}_{n,n} - \alpha_I W')^{-1}$ is the Leontief matrix, the superscript $\tilde{.}$ is used for logarithmic values, $U := (u_{1,t}, ..., u_{n,t})'$ is the industries' log productivity vector, $Z_t := (\zeta_{1,t}, ..., \zeta_{n,t})'$ the industries' spending vector, $\zeta_t := \zeta_{1,t} + ... + \zeta_{t,n}$ is the aggregate industries' spending and $A := (a_i)_{i=1,...,n}$ is a constant vector with $a_i := \alpha_I \tilde{\alpha} - \alpha_I \tilde{\alpha}_I - \alpha_I \sum_{j=1}^n w_{ij}\tilde{w}_{ij}$.

Furthermore, the final good price is determined by nominal output y_t^N, capital k_{t-1}, labour supply l_t and the scalar linear combination of shocks $u_t = \sum_{i=1}^n \tau_i u_{i,t}$, where by definition, $T := (\tau_1, ..., \tau_n)' := \mathscr{L}\Theta$

$$\tilde{p} = (\mathscr{L}\Theta)'A + \frac{1}{1-\alpha_I}\left\{ (\alpha_K + \alpha_L)\tilde{\zeta}_t - \alpha_K \tilde{k}_{t-1} - \alpha_L \tilde{l}_t \right\} + (\mathscr{L}\Theta)' \left[R\tilde{E}V_t - \alpha \tilde{Z}_t \right] - u_t. \tag{3.3.4}$$

Proof. The proof is presented in appendix 3.A.1. ■

At this stage, it is useful to make an assumption regarding the utility function in order to obtain an explicit labour supply condition.

Assumption 1. *Assume that the utility function is given by*

$$\mathscr{U}_t = \ln(c_t) + \frac{\psi}{1-\nu}\left(\frac{M_t^H}{P_t}\right)^{1-\nu} - \frac{\chi}{1+\eta} l_t^{1+\eta}, \tag{3.3.5}$$

where ν is the curvature on the utility form holding real cash balances, η the curvature on dislike of labour and χ is a parameter that controls for households' dislike of labour. By convention, if $\nu = 1$, the utility function becomes

$$\mathscr{U}_t = \ln(c_t) + \psi \ln\left(\frac{M_t^H}{P_t}\right) - \frac{\chi}{1+\eta} l_t^{1+\eta}. \tag{3.3.6}$$

Under assumption 1, labour provision is driven by the equation

$$\chi l_t^\eta = \frac{\omega_t}{p_t}\frac{1}{c_t}. \tag{3.3.7}$$

Combine this labour supply condition with the intermediate producers' labour first-order condition to obtain an expression of labour supply and prices as per the corollary below.

Corollary 1. *Under assumption 1, labour provision is given by*

$$\tilde{l}_t = \frac{1}{\eta+1}(\tilde{\alpha}_L - \tilde{\alpha} - \tilde{\chi} + \tilde{\zeta}_t - c_t^{\tilde{N}}), \tag{3.3.8}$$

and the good prices are given by

$$\tilde{P}(t) = \mathscr{L}' \left\{ \hat{A} + \left[\left(\alpha_K + \frac{\eta}{1+\eta}\alpha_L \right) \tilde{\zeta}_t - \alpha_K \tilde{k}_{t-1} + \frac{\alpha_L}{1+\eta}\tilde{c}_t^N \right] \mathbf{1}_{n,1} + R\tilde{E}V_t - \alpha \tilde{Z}_t - U_t \right\}, \tag{3.3.9}$$

where $\hat{A} := A + \frac{\alpha_L}{1+\eta}(-\tilde{\alpha}_L + \tilde{\alpha} + \tilde{\chi})$ and $\tilde{c}_t^N := p_t c_t$ is consumption spending. The final good price is

$$\tilde{p}_t = \frac{1}{1-\alpha_I} \left\{ \left(\alpha_K + \frac{\eta}{1+\eta}\alpha_L \right) \tilde{\zeta}_t - \alpha_K \tilde{k}_{t-1} + \frac{\alpha_L}{1+\eta}\tilde{c}_t^N \right\} + (\mathscr{L}\Theta)' \left[\hat{A} + R\tilde{E}V_t - \alpha \tilde{Z}_t \right] - u_t. \tag{3.3.10}$$

Proof. The proof is presented in appendix 3.A.1. ■

To simplify the model solution, I make the following assumption on the economy's agents behaviour with regard to potential future underspending.

Assumption 2. *The economy's agents do not take future firms' underspending into account when making their saving and financing decisions.*

As we have seen in the previous chapter, this assumption is verified in approximation when nominal interests rates are not too close to the zero lower bound. Assumption 2 implies a simpler reformulation of the intermediate producers' financing equations

$$\mathbb{E}_t\left[\alpha rev^C_{i,t+1}\right] = r^F_t \frac{\iota_{i,t}}{1+x^M_t} \text{ for } i = 1, ..., n, \tag{3.3.11}$$

where $rev^C_{i,t+1} := p_{i,t+1}x_{i,t+1}$ denotes the next period's revenues of industry i, assuming that all industries' spending constraints are binding, regardless of next period's shocks. Alternatively, one can rewrite the financing conditions 3.2.34, by exploiting the revenues' expression in lemma 1, as follows

$$r^F_t I_t = \mathbb{E}_t\left[\alpha_I W' I_t + \alpha(1+x^M_t)y^{N,C}_{t+1}\Theta\right] \text{ for } i = 1, ..., n, \tag{3.3.12}$$

where $y^{N,C}_{t+1}$ denotes the next period's nominal output, assuming that all industries' spending constraints are binding, regardless of next period's shocks. Inverting the equation above yields an expression for industry financings as a function of expected nominal output as shown in the proposition 4.

Proposition 4. *Under assumption 2, one can write the financing conditions as follows*

$$I_t = \frac{\alpha}{r^F_t}(1+x^M_t)\mathbb{E}_t[y^{N,C}_{t+1}]\Gamma_t\Theta, \tag{3.3.13}$$

where by definition $\Gamma_t = \left(\mathbb{I}_{n,n} - \frac{\alpha_I}{r^F_t}W'\right)^{-1}$ *and* $\Theta = (\theta_1, ..., \theta_n)'$.

Under both assumption 1 and 2, the households' demand for saving through deposits and physical capital are driven by the equations

$$\frac{1}{c^N_t} = \frac{\beta r_t}{1+x^M_t}\mathbf{E}_t \frac{1}{c^{N,C}_{t+1}}, \tag{3.3.14}$$

$$\frac{1}{c_t} = \mathbf{E}_t \frac{1}{c^C_{t+1}}(1-\delta+r^{K,C}_{t+1}), \tag{3.3.15}$$

where the next time period's variables denoted by the ^{C} correspond to a version of the model where all industries are constrained in the next time period, regardless of the next period's shocks.

Proof. Equation 3.3.13 results from inverting equation 3.3.12. Equations 3.3.14 and 3.3.15 derive immediately from the housholds' Euler intertemporal equations under assumptions 1 and 2. ∎

The corollary below is derived immediately from proposition 4, by noting that the sum of the elements of the vector $\Gamma_t\Theta$ is $\frac{r_t^F}{r_t^F-\alpha}$.[4]

Corollary 2. *If the overall financing in the economy is noted $\iota_t = \iota_{1,t} + ... + \iota_{1,t}$, then under assumption 2, each industry's financing is obtained as follows*

$$\iota_{i,t} = \gamma_{i,t}\left(1 - \frac{\alpha_I}{r_t^F}\right)\iota_t, \tag{3.3.16}$$

$$\iota_t = \frac{\alpha}{r_t^F - \alpha_I}(1 + x_t^M)\mathbb{E}_t[y_{t+1}^{N,C}], \tag{3.3.17}$$

where by definition $(\gamma_{1,t}, ..., \gamma_{n,t})' := \Gamma_t\Theta$.

These results imply that in any simulation of the model, the n state variables $\iota_{1,t}, ..., \iota_{n,t}$ can be replace by ι_t and r_t^F. In addition, note how higher financing costs decrease financing and therefore bring future spending lower (equation 3.3.17). Financing costs introduce a wedge between the marginal product of inputs and their real costs. Moreover, note that the fraction of aggregate spending allocated to industry i is $\gamma_{i,t}\left(1 - \frac{\alpha_I}{r_t^F}\right)$ and depends on the financing cost facing firms (equation 3.3.16). In the absence of financing costs, the fraction of aggregate financing going to industry i would be $\tau_i(1-\alpha_I)$. The working capital friction can affect the allocation of resources across industries even when all firms face the same financing cost. Furthermore, the size of the financing/spending reallocation effect depends on the input-output structure. This behaviour is consistent with the findings in Bigio and LaO (2020).

[4] This is achieve by writing $\mathbf{1}_{1,n}.\Gamma_t\Theta = \mathbf{1}_{1,n}.\sum_{k=0}^{\infty}\left(\frac{\alpha_I}{r_t^F}\right)^k W'\Theta$ and noting that $\mathbf{1}_{1,n}.W'\Theta = 1$ for all k.

The next section studies the effects of the input-output relationships and the final production function parameters $\theta_1, ..., \theta_n$ on the model's mechanism and presents some industry aggregation results.

3.4 Effect of input-output relationships

The proposition below provides a criteria to determine which set of industries, if any, relax their spending below the financing constraint.

Proposition 5. *There are exactly m industries $i = 1, ..., m$ where spending is unconstrained by the previously set financing, if and only if*

$$\max_{i=m+1,...,n;\eta_i^m \neq 0} \frac{\delta_i^m(I_{m+1:n,t-1})}{\eta_i^m} < \alpha(1 + x_{t-1}^M)y_t^N \leq \min_{i=1,...,m;\tau_i^m \neq 0} \frac{\nu_i(I_{t-1})}{\tau_i^m}, \tag{3.4.1}$$

where

$$T_m := (\tau_1^m, ..., \tau_m^m)' := \mathscr{L}_m \Theta_{1:m},$$

$$(\eta_{m+1}^m, ..., \eta_n^m)' = \alpha_I W'_{m+1:n,1:m} T_m + \Theta_{m+1:n},$$

$$(\delta_{m+1}^m, ..., \delta_n^m)' = I_{m+1:n,t-1} - (\alpha_I^2 W'_{m+1:n,1:m} \mathscr{L}_m W'_{1:m,m+1:n} + \alpha_I W'_{m+1:n,m+1:n}) I_{m+1:n,t-1},$$

$$(\nu_1^m, ..., \nu_m^m)' = I_{1:m,t-1} - \alpha_I \mathscr{L}_m W'_{1:m,m+1:n} I_{m+1:n,t-1}$$

and $\mathscr{L}_m := (\mathbb{I}_{m,m} - \alpha W'_{1:m,1:m})^{-1}$ is the Leontief matrix corresponding to the subset of unconstrained industries $i = 1, ..., m$.

Proof. The proof is presented in appendix 3.A.3. ∎

Note that the constants $\tau_i^m, ..., \tau_m^m$ and $\eta_{m+1}, ..., \eta_n$ only depend on the input-output structure and the final production function parameters whilst the variables δ_i^m and ν_i^m also depend on the previous period industries' financing I_{t-1}. Note also, that Independently of the level of shocks and their influence on nominal output, a feasible set of unconstrained industries $i = 1, ..., m$ needs to verify $\max_{i=m+1,...,n} \frac{\delta_i^m(I_{m+1:n})}{\eta_i} < \min_{i=1,...,m} \frac{\nu_i(I_{m+1:n})}{\tau_i^m}$. Because the constant

τ_i and η_i^m are only a function of the model's parameters and the terms δ_i^m, ν_i and x_{t-1}^M are predetermined at time t, the only contemporaneous variable deciding the subset of unconstrained industry is the aggregate nominal output y_t^N. This implies that industry shocks matter to underspending only in the way that they determine the aggregate nominal output given the model's state variables.

In order to study the behaviour of the model's main mechanism when the number of industries making the productive sector becomes large, I focus on the situation where all industries enter an unconstrained spending mode. The proposition below provides a sufficient and necessary condition for every industry to spend less than the financing constraint.

Proposition 6. *All industries are facing relaxed financing constraints if and only if*

$$y_t^N \leq \frac{1}{\alpha(1+x_{t-1}^M)} \min_{\tau_i \neq 0} \frac{\iota_{i,t-1}}{\tau_i}, \tag{3.4.2}$$

where the constants τ_i *are non-negative and are defined as:* $(\tau_1, ..., \tau_n)' := \mathscr{L}\Theta$, *with* $\mathscr{L}$ *being the Leontief matrix* $\mathscr{L} := (\mathbb{I}_{n,n} - \alpha_I W')^{-1}$ *and* $\Theta := (\theta_1, ..., \theta_n)'$ *a vector representing final production.*

Under assumption 2, all the industries would relax their financing constraints if and only if nominal output y_t^N *verifies*

$$\frac{y_t^N}{\mathbf{E}_{t-1} y_t^{N,C}} \leq \frac{1}{r_{t-1}^F} \min_{\tau_i \neq 0} \frac{\gamma_{i,t-1}}{\tau_i} \tag{3.4.3}$$

where by definition $(\gamma_{1,t-1}, ..., \gamma_{n,t-1})' := \Gamma_{t-1}\Theta$ *and* $\Gamma_{t-1} := \left(\mathbb{I}_{n,n} - \frac{\alpha_I}{r_{t-1}^F} W'\right)^{-1}$ *and* $y_t^{N,C}$ *denotes the period* t *nominal output assuming that all industries' spending constraints are binding as per assumption 2.*

Proof. The proof of the proposition is in Appendix 3.A.3. Condition 3.4.2 follows immediately from proposition 5. ∎

The first result of proposition 6 links the event where the whole economy enters an underspending mode to the levels of raised financing and to the constants τ_i that are determined by final production as summarised by the vector $\Theta := (\theta_1, ..., \theta_n)'$ and the matrix

$\alpha_I W := \alpha_I (w_{i,j})_{i,j=1,...,n}$ summarising the input-output relationships. As one might expect, higher financing levels $\iota_{i,t-1}$ make it more likely for all industries to enter an underspending mode. On the other hand, one can prove that in a model where the firms do not require financing before spending, industry i produces a fraction $(1-\alpha_I)\tau_i$ of the overall intermediate output in the steady-state. This means that the ratio $\iota_{i,t-1}/\tau_i$, that is key to condition 3.4.2, can be interpreted as relating the financing allocated to industry i to the proportion of steady-state output produced by the same industry in the absence of the firms' need to finance before spending on production.

Condition 3.4.3 shows more explicitly the effect of the cost of financing r^F_t. It is important to note that the terms $\gamma_{i,t-1}$ are non-increasing in r^F_{t-1}.[5] This implies that a lower cost of financing makes it more likely it is for all industries to be unconstrained by their financing levels for a given distribution of nominal output. The left-hand side of condition 3.4.3 is the ratio of nominal output over the nominal output expected by producers in the previous period. This condition states that all industries underspend when nominal output drops below previous agents' expectations by an amount that depends on the previous financing rate and the economy's input-output structure.

Assumption 2 is key to obtain condition 3.4.3 from 3.4.2. As seen in chapter 2, this assumption is verified in approximation when the net financing costs are not too close to the zero lower bound. To see this, first note that the term in the right-hand side of equation 3.4.3 is one if the gross financing rate is one and that this term decreases with r^F_T (as noted earlier) and converges towards zeros for high values of r^F_T. This implies that as net financing rates get farther from the zero lower bound, the term $\frac{1}{r^F_{t-1}} \min_{\tau_i \neq 0} \frac{\gamma_{i,t-1}}{\tau_i}$ gets closer to zero making underspending unlikely under assumption 2 (baring an infinite volatility of nominal output, which any decent model calibration would exclude). In other words, if financing rates are high enough, underspending becomes very unlikely under assumption 2. Now, note that financing is higher under assumption 2 compared to a situation where agents take future underspending into account by comparing the financing equations 3.3.11 and 3.2.34. Given that lower financ-

[5] Note that $(\gamma_{1,t-1}, ..., \gamma_{n,t-1})' := \Gamma_{t-1}\Theta = \sum_{k=0}^{\infty}(\alpha_I/r^F_{t-1})W'^k\Theta$ and that the matrix W' and the vector Θ are non-negative.

ing implies that underspending is even less likely, one can conclude that assumption 2 is true in approximation if the financing rate is high enough. When studying particular input-output networks below, we will see that financing rates do not need to be very high for underspending to become unlikely.

The aggregate variables of the model presented here is driven by the aggregate log-productivity shock $u_t = \sum_{i=1}^{n} \tau_i u_{i,t}$. The volatility of u_t can be written in the following form

$$\sigma^u := \sqrt{\Theta' \mathscr{L}' V \mathscr{L} \Theta}, \tag{3.4.4}$$

where V is the variance/covariance matrix of the industry shocks e_i. This formulation enables to see that potential positive correlation of industry shocks would increase the volatility σ^u and therefore make the model aggregate variables more volatile. In the current setup V has zero off diagonal elements. To simplify further, assume that: $\sigma_i = \sigma$ for $i = 1, ..., n$. Then the formula for σ^u simplifies as follows:

$$\sigma^u = \sigma \sqrt{\sum_{i=1}^{n} \tau_i^2}, \tag{3.4.5}$$

with the constants τ_i being the element of the vector $T := (\tau_i)_{i=1,\ldots,n} := \mathscr{L}\Theta$.

The vector T comprises the effect of both the final production parameters Θ and the input-output matrix W on the volatility σ^u. To fix ideas, let us first study the effect of the parameters $\theta_1, ..., \theta_n$. To that effect, assume that all industries are disconnected: $W = \mathbb{I}_{n,n}$. Then the value of σ^*_{cs} would be

$$\sigma^u = \frac{\sigma}{1-\alpha_I} \sqrt{\sum_{i=1}^{n} \theta_i^2}. \tag{3.4.6}$$

In the case where final production only uses a single good: $\theta_1 = 1$ and $\theta_i = 0$ for $i > 1$, the volatility of consumer spending is maximal $\sigma^u = \frac{\sigma}{1-\alpha_I}$ while it is minimal $\sigma^u = \frac{\sigma}{1-\alpha_I}\frac{1}{\sqrt{n}}$, when all goods play the same role in final production $\theta_i = 1/n$ for $i = 1, .., n$. Everything else being equal, an economy where the inputs to the production of the consumption good are spread across many intermediate goods would have a lower aggregate fluctuations and the mechanism of the model would be less likely to operate.

To isolate the impact of the input-output network on aggregation, one can assume that all intermediate goods play the same role in final production ($\theta_i = 1/n$). This puts us in the setup presented in Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and leads to the same conclusion, namely that fluctuations are higher for networks characterised by dominant industries that play an asymmetric role as a supplier to other sectors. An example of such networks is the "star" production network. I refer the reader to the aforementioned paper for more details on the role played by networks in aggregating industry-specific shocks and turn to network effects in the context of this chapter's main mechanism.

To assess the effect of the input-output matrix, I assume that $\theta_i = 1/n$ for $i = 1, ..., n$ and study the aggregation of the underspending mechanism under two notable networks: the symmetric fully connected network and the star network. To simplify the analysis further, I make the following assumption regarding the money in the utility function parameter ν for the remainder of this section. This assumption is standard in the literature, and will be maintained when simulating the model in section 3.5.

Assumption 3. $\nu = 1$.

As explained in appendix 3.A.2 (proposition 8), this assumption guarantees that the steady-state financing rate $\bar{r}^F$ is independent of the number of industries n and the input-output matrix W. This result will be useful when studying aggregation in the two notable networks mentioned above.

Symmetric fully connected network: In the case where all industries play a symmetric role in the economy and are all equally interconnected ($w_{ij} = 1/n$, $\theta_i = 1/n$), the constants τ_i are given by

$$\tau_i = \frac{1}{(1-\alpha_I)n} \text{ for } i = 1, ..., n. \tag{3.4.7}$$

This implies that the volatility of the aggregate shock u_t is

$$\sigma^u = \frac{\sigma}{1-\alpha_I}\frac{1}{\sqrt{n}}. \tag{3.4.8}$$

Note how the volatility terms is close to zero for large numbers of industries n. Assuming the fully connected network structure the terms $\gamma_{i,t-1}$ are as follows

$$\gamma_{i,t-1} = \frac{1}{(1-\alpha_I/r^F_{i,t-1})n} \text{ for } i = 1, ..., n. \tag{3.4.9}$$

In this case, the allocation of aggregate financing to each industry $\gamma_{i,t-1}(1-\alpha_I/r^F_{i,t-1}) = 1/n$ is independent of the loan rate r^F_{t-1}. In other words, the working capital friction does not affect the allocation of resources across industries. The expressions of τ_i and γ_i enable us to rewrite the necessary and sufficient condition for all industries to be unconstrained in proposition 6 as follows

$$\frac{y^N_t}{\mathbf{E}_{t-1}y^{N,C}_t} \leq \frac{1-\alpha_I}{r^F_{i,t-1}-\alpha_I} = 1 - \frac{r^F_{i,t-1}-1}{r^F_{i,t-1}-\alpha_I}. \tag{3.4.10}$$

For all industries to reduce their spending below the level provided by previous financing, nominal output needs to drop below the firms' previous period expectations by more than $\frac{r^F_{i,t-1}-1}{r^F_{i,t-1}-\alpha_I}$. Under assumption 3, the steady-state financing rate $\bar{r}^F$ is independent of the number industries n. Given that the aggregate volatility σ^u becomes small for large values of n, while the required drop of nominal output is stable, this indicates that the underspending mechanism becomes unlikely to operate under the fully connected network when the intermediate production sector is composed of a very large number of industries.

Star network: On the other hand, if one assumes that one industry (let's say industry 1) is the only provider of intermediary input to all industries and that all intermediate goods play the same role in final production ($w_{ij} = 1$ if $j = 1$ and $w_{ij} = 0$ otherwise, $\theta_i = 1/n$ for all i),

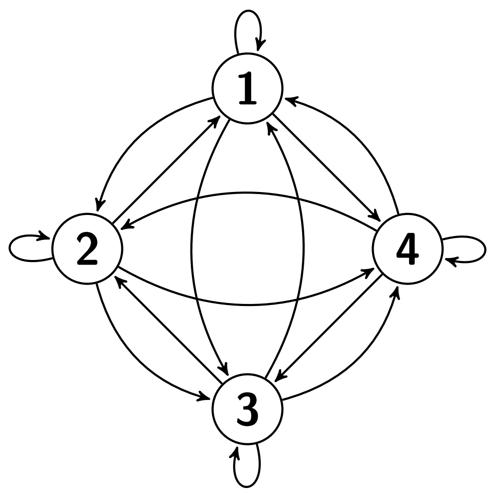

Figure 3.1: Fully connected network in an economy with 4 industries. Arrows show the direction of input provision: they depart from the industry providing the intermediary input and arrive to the industry using the input.

one can derive the expressions below for the constants τ_i and the terms $\gamma_{i,t-1}$

$$\tau_1 = \frac{1}{n} + \frac{\alpha_I}{1-\alpha_I}, \tag{3.4.11}$$

$$\tau_i = \frac{1}{n} \text{ if } i > 1, \tag{3.4.12}$$

$$\gamma_{1,t-1} = \frac{1}{n} + \frac{\alpha_I}{r^F_{t-1} - \alpha_I}, \tag{3.4.13}$$

$$\gamma_{i,t-1} = \frac{1}{n} \text{ if } i > 1. \tag{3.4.14}$$

In the context of the star network described here, the industry 1 is often referred to as the source industries, while the other industries are referred to as sink industries. It is noteworthy that the fraction of aggregate financing allocated to the source industry is given by $\gamma_{i,t-1}(1-\alpha_I/r^F_{i,t-1}) = 1/n + (\alpha_I/r^F_{t-1})(1-1/n)$ and decreases with cost of financing r^F_{t-1}. Unlike in the case of symmetric network studied above, allocation across industries is impacted by the working capital friction when production relations are represented by a star network. Higher financing costs have a similar effect to an increase of the cost of all inputs, including the cost of the intermediate input produced by the source industry. This reduces demand for the output of the source industry thus reducing the proportion of aggregate financing dedicated to it.

The formulae of the terms τ_i and $\gamma_{i,t}$ above imply an aggregate volatility σ^u that remains

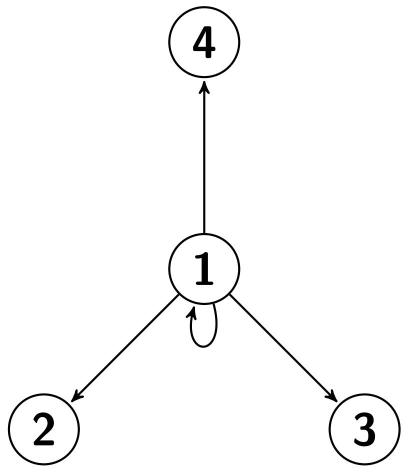

Figure 3.2: Star network in an economy with 4 industries. Arrows show the direction of input provision: they depart from the industry providing the intermediary input (source industry) and arrive to the industry using the input (sink industries).

stable and positive for very large values of n

$$\sigma^u = \sigma\sqrt{(n-1)\frac{1}{n^2} + \left(\frac{1}{n} + \frac{\alpha_I}{1-\alpha_I}\right)^2} \approx \frac{\alpha\sigma}{1-\alpha} \text{ for } n >> 1. \tag{3.4.15}$$

Assuming a star network, the condition 3.4.3 for all industries to be in an unconstrained spending mode becomes

$$\frac{y_t^N}{\mathbf{E}_{t-1}y_t^{N,C}} \leq \frac{1}{r_{i,t-1}^F}\frac{1+\alpha_I n/(r_{i,t-1}^F - \alpha_I)}{1+\alpha_I n/(1-\alpha_I)} \approx \frac{1}{r_{i,t-1}^F}\frac{1-\alpha_I}{r_{i,t-1}^F - \alpha_I} \text{ for } n >> 1. \tag{3.4.16}$$

Under assumption 3, the steady-state loan rate $\bar{r}^F$ is independent of number of industries n. This implies that, when the economy is previously in the steady-state, the drop in nominal output required for all industries to enter an underspending mode remains stable as the number of industries changes. In addition, the volatility σ^u converges to a positive level as the n grows large ($\sigma^u \approx \frac{\alpha\sigma}{1-\alpha}$). One can therefore conclude that, under a star network, the underspending mechanism can function even when the intermediate production sector is composed of a large number of industries.

In the analysis above, I use the behaviour of σ^u for large n as a proxy for the volatility of y^N as the number of industries n grows large. Because the model equations also change when n changes, this reasoning requires further justification. Figure 3.3 shows the behaviour of the impulse response functions of the variables y^N and c^N as n grows larger, assuming the fully connected network and the star network, respectively. The figure confirms the analysis above, with the impulse response of y^N and c^N converging towards the zero line in the case of the fully connected network and towards a stable non nil response for the star network. As shown above, the unexpected drop in nominal output required for all industries' spending to be unconstrained is stable for large n for both types of networks. This implies that the probability of all industries entering an underspending mode converges towards a positive value in the case of a star network, and towards zero in the case of the fully connected network.

The results of this section point towards the fact that input-output production networks where a small number of industries play an important role in providing intermediary inputs would imply that the mechanism of the model is more likely to operate when the number of independent industries is large. These results are consistent with and guided by existing literature, for instance Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and extend a literature mainly concerned with the network origins of aggregate output volatility in a direction where part of the asymmetries affecting the distribution of aggregate variables can also be attributed to the nature of the production networks prevailing in the economy.

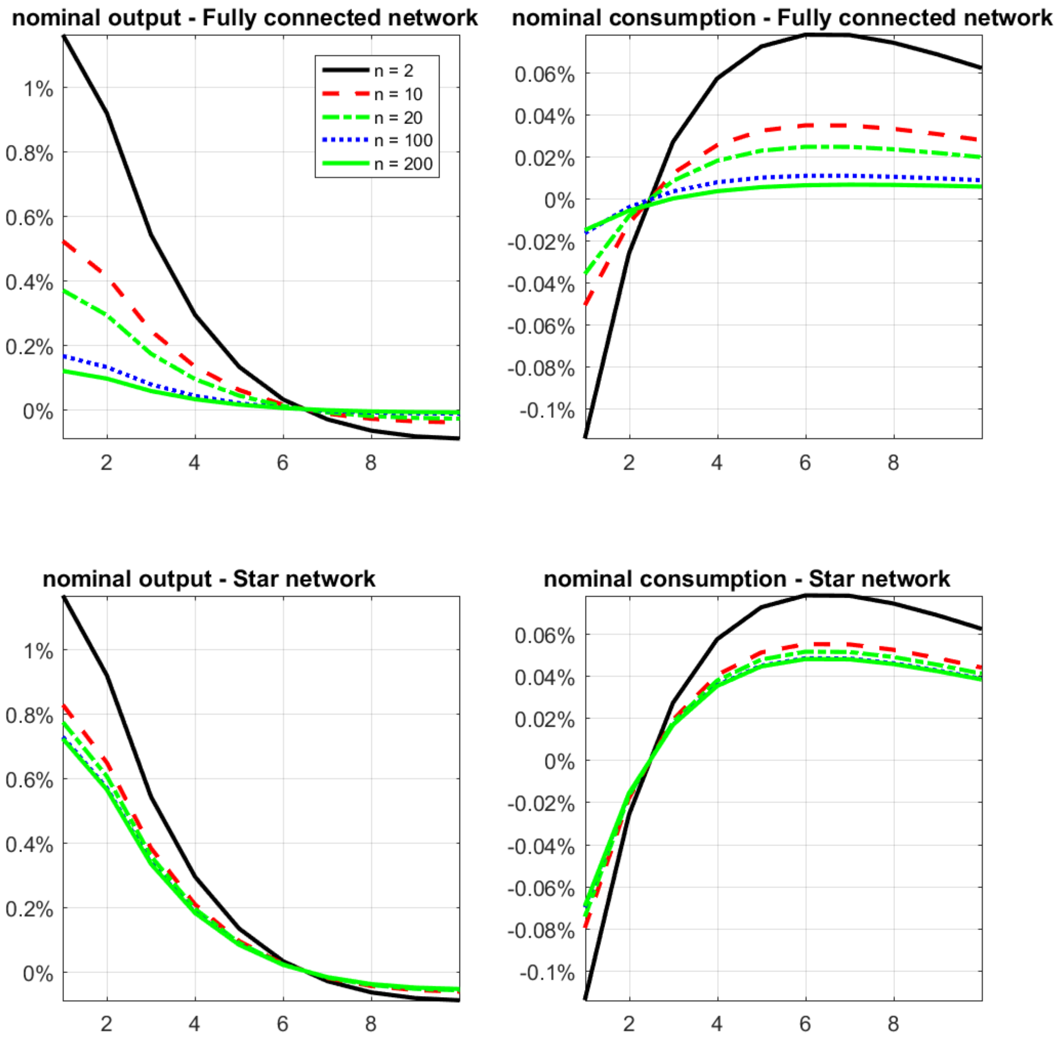

Figure 3.3: Impulse response functions of the nominal output y^N and nominal consumption c^N following a shock to industry 1, assuming the fully connected network and the star network and different numbers of industries n. The size of the industry 1 shock is chosen to imply a one standard deviation move of the aggregate shock variable u_t. The remaining model parameters are calibrated as in section 3.5. All variables' responses are presented in a logarithmic form and as a deviation from the steady state.

3.5 Simulation results

3.5.1 Simulation routine

To devise a simulation methodology, I first note that, whatever the number of intermediate industries, the financing state variables $\iota_{1,,t-1}, ..., \iota_{n,t-1}$ can be replaced by just two state variables: the aggregate financing $\iota_{t-1} = \iota_{1,t-1} + ... + \iota_{n,t-1}$ and the financing rate r^F_{t-1} (corollary 2). Then, I note that the majority of the model variables can be expressed as a function of the nominal output y^N_t and consumption spending c^N_t (proposition 1, lemma 1, corollary 1 and proposition 4).

In the absence of forward-looking equations in the model, one can solve for the values of y^N_t and c^N_t that satisfy all the model equations presented above. The main difficulty consists in dealing with the model's forward-looking equations. These are the households' capital and deposit savings equations 3.3.14 and 3.3.15, and the intermediate producers' financing equations 3.3.13. Given assumption 2, the behaviour of the next period's variables necessary to estimate the expectations terms in the model equations can be modelled using a version of the model where the spending constrained are always binding. This can be achieved through standard perturbation techniques around the model's steady-state.[6] Perturbation near the steady-state helps provide a policy function for the variables $y^{N,C}_{t+1}$, $c^{N,C}_{t+1}$, c^C_{t+1} and $r^{K,C}_{t+1}$, conditional on the period's t state variables and the period $t+1$ exogenous model shocks, thus providing a way to approximate the expectations terms in the model equations through quadrature.

The model is then simulated by writing all the model equations, directly or indirectly, as a function of y^N_t and c^N_t as explained above and solving for the values of y^N_t and c^N_t such as all the model equations are verified. The model equations involving no expectations over the behaviour of future model variables are solved exactly, while those involving future model variables are solved through quadrature using the policy functions obtained through perturbation around the steady-state, as explained above.[7] Like the "Exact Today" algorithm presented in Den Haan, Kobielarz, and Rendahl (2016), this period's outcomes are an exact solution of the model equations, and I only approximate the next period's outcomes. In Den Haan, Kobielarz, and Rendahl (2016), however, the policy functions determining the next period's outcomes are approximated through perturbation around the current state of the model at each point of the simulation path. This helps with solving models that do not possess a steady-state and

[6]Using the numerical suite DYNARE, for example.

[7]The steady-state of the model where the spending constraint always hold is the same as the steady-state of the main model (see appendix 3.A.2).

provides a way to incorporate the effects of uncertainty and non-linearities and interactions between these two effects. As shown in appendix 3.A.2, the model studied here does possess a unique steady-state. Furthermore, assumption 2 implies little interaction between uncertainty over the future and the non-linearities emanating from the underspending mechanism. This justifies the choice of using policy functions obtained through perturbations around the steady-state to approximate the next period's outcome. A detailed description of the algorithm used is presented in appendix 3.B.2. The appendix also provides simulations testing the accuracy of the suggested simulation routine.

3.5.2 Calibration

The model is studied in the symmetric fully connected case ($w_{ij} = 1/n$ and $\theta_i = 1/n$ for all i, j) for an economy with two industries ($n = 2$) and is simulated yearly. Households are assumed to have the utility in assumption 1. The utility discount factor β helps determine the equilibrium rates of financing and is, therefore, key to the main mechanism of the model. Mehra and Prescott (1985) report that between 1889 and 1978, the average annual real return on equity was 7%, while the average annual real return on short term debt was 1%. In the current setup, I assume an investment horizon of one year. I then target an equilibrium real interest rate closer to the return on short term debt at 2%. This implies that $\beta = 0.98$. The steady-state money growth $\bar{x}^M = 0.02$ matches a steady-state inflation at 2%. Following Atkenson, Khan, and Ohanian (1996), I assume moderate decreasing return to scale in the intermediate production sector[8] $\alpha = 0.95$. Following Basu (1995) and Jones (2011a), I assume that the share of intermediate input is $\alpha_I = \alpha/2 = 0.475$. The share of capital is assumed to be half the share of labour, which yields $\alpha_K = 0.1583$ and $\alpha_L = 0.3167$. The calibration is performed to match a steady-state where the level of employment of $\bar{l} = 0.3$ and where households hold a quarter of the money stock $\bar{m}^H = 0.25$ (matching the M1/M2 ratio for the United States). This determines the value of the parameters χ and ψ.[9] The industry TFP standard deviations ($\sigma_1 = \sigma_2$) are such as aggregate TFP is around 1%. The remaining model parameters are

[8] See also Veracierto (2001) and Atkeson and Kehoe (2005).

[9] Refer to appendix 3.A.2 for more details regarding the determination of the steady-state.

Households' utility parameters	
χ	8.6
η	1
ψ	0.03
ν	1
β	0.98
Intermediate production parameters	
α_I	0.475
α_K	0.1583
α_L	0.3167
δ	0.1
Exogenous shocks parameters	
σ_i	0.0282
ρ	0.7
σ^M	0.005
ρ^M	0.5
$\bar{x}^M$	0.02

Table 3.1: Assumed and calibrated model parameters.

standard and are borrowed from the literature. Table 3.1 shows the assumed and calibrated model parameters.

3.5.3 Dynamic effects of firms' underspending

Figures 3.4 compares impulse responses of the model with the underspending mechanism to a version of the model whereby the financing constraints are always binding (benchmark). The figure shows the impulse responses of several aggregate variables following a large negative shock to productivity in industry 1 ($-4 \times \sqrt{2}$ standard deviations), while keeping productivity in industry 2 unchanged. The industry 1 shock size is chosen to imply an aggregate output drop around 8% in the benchmark model, which approximately corresponds to the drop of U.S. GDP below its trend between Q2 2008 and Q2 2009.

When the underspending mechanism operates, the negative productivity shock has a larger immediate effect on output. When they are allowed to underspend, firms adapt faster after gaining knowledge of the new lower productivity by reducing expenditure before adjustment to financing levels affects the economy. Figure 3.4 shows that the underspending mechanism causes output to drop by a further 1% and capital investments to drop by a further 4%, whilst making labour drop by an extra 1.5%. When spending by firms is curtailed, less cash is dedicated to paying wages. Lower wages push the elastic labour supply lower in the presence of underspending, which in turn causes an additional drop in output. Lower output implies lower investments in physical capital, as households decrease their savings to smooth consumption.

On the other hand, when the firms cannot adjust expenditure, their previous level of financing, which was based on a more optimistic view of productivity, helps dampen the severity of the current negative shock. In particular, the spending on wages remains stable, which implies a relatively stable labour supply. In both models, the reverse hump shape reaction of consumption implies lower deposit rates. This reaction is, however, muted by the rather smooth change in households' consumption. Assumption 2 implies no difference in the dynamics of financing between the main model and the benchmark model. One period after the unexpected productivity shock, there is no underspending in either model. This and the identical reaction of financing in the period where the surprise shock occurs imply that the impulse responses of both models are very similar beyond the first time period.[10]

Figures 3.5 and 3.6 show the impulse response of several industry-specific variables. Let us first consider the reaction of the benchmark model to the unexpected negative productivity shock to industry 1. Following the shock, the output of industry 1 drops as expected. The price of the scarcer good 1 increases, thus the increasing the production cost of industry 2 firms. At the same time, the increase of the good 1 price pushes firms in both industries as well as the final producer to divert more demand toward industry 2 (substitution effect). The higher demand effect partially compensates for the effect of higher production costs, and the output of industry 2 witnesses a small drop as a result. In the main model, the less productive industry 1 firms adjust their spending lower following the large unpredicted shock affecting their productivities. The underspending in industry 1 decreases the labour supplied by households to this industry, thus making its output drop lower relative to the benchmark model. In addition, underspending in industry 1 pushes the demand for good 2 lower, thus incentivising industry

[10]The identical reaction of financing in both models is a consequence of assumption 2.

2 firms to underspend. The underspending by industry 2 firms decreases the labour employed by this industry, and this industry's output drops further as a result. Underspending causes both industries' contribution to the final production to drop by an extra 1%. Substitution effects operating at the level of final and intermediate producers mean that, in the absence of underspending, the industry 2 output and its contribution to final production would have been more stable than the output and contribution of industry 1. This relative stability of output within industry 2 makes the impact of the model's main mechanism more visible in this industry where productivity remains unchanged.

As mentioned above, the reaction of households to the lower wage spending by firms is key to the functioning of the underspending mechanism and its effects on aggregate variables. The reaction of labour supply to changes to firms' wage spending depends on the assumed level of Frisch elasticity of labour supply $1/\eta$. Figure 3.7 provides the impulse response of aggregate variables to an unpredicted shock to industry 1 productivity when a higher Frisch elasticity is assumed (lower $\eta = 0.1$). Assuming a higher Frisch elasticity of labour implies that firms' underspending makes output drop by a further 2.5% (as opposed to 1% for $\eta = 1$). This is caused by a larger drop in labour supply in the presence of the underspending mechanism (4.2% instead of 1.5% in the main calibration). The higher elasticity of labour supply causes a moderate increase in underspending in both industries (figures 3.8 and 3.9). This moderate increase in underspending is compounded by the higher elasticity of labour supply, making underspending have a larger negative effect on labour hours. The larger negative impact of underspending on hours implies a larger negative impact on output. In addition, in the new calibration ($\eta = 0.1$), investment drops by an extra 9% in the presence of the underspending mechanism (instead of 4% in the main calibration). This picture is confirmed when looking at industries 1 and 2 output and contributions to final production. All these quantities drop more when the elasticity of labour supply is higher.

In order to assess the dynamic effects of the nature of the input-output network, I present the impulse responses of both the benchmark and the main model to an unexpected productivity shock affecting a single industry, assuming an economy with two industries connected through a star network ($n = 2$ and $w_{i1} = 1, w_{i2} = 0$ for $i = 1, 2$ and $\theta_1 = \theta_2 = 1/2$.).[11] A 4.24× standard deviations negative shock is assumed to affect the industry that produces all intermediary goods in the economy (industry 1 or the source industry), and productivity is assumed to remain stable in the sink industry (industry 2). The impulse responses of the model aggregate and industry-specific variables are presented in figures 3.10, 3.11 and 3.12. As in the fully connected network case, the size of the productivity shock to industry 1 is calibrated to imply an 8% drop in output. Let us first note that, in the context of the star network, the size of the source industry productivity shock required to achieve an 8% drop of output in the benchmark model is smaller than the size of the single industry shock required when a fully connected network describes inter-industry relations as above. This is in line with the theoretical results of section 3.4. Assuming a star network and a smaller shock to industry 1, the response of aggregate variables is strikingly similar to the response in the case of a fully connected network. A different picture emerges when considering the impulse responses of each industry, as the asymmetry of the star network causes a differentiated behaviour across industries (figures 3.11 and 3.12). Because, under the star network, the sink industry output is solely used as input in the final production process and is not used as input by the source industry, the drop in the sink industry output is matched by the drop of the same industry's contribution to final production. Lower spending in the source industry affects its output through the direct labour supply channel. The decrease in the source industry output due to underspending increases the production costs of the sink industry, pushing its output lower. This means that industry 2 suffers from its own underspending, through the direct labour supply channel and from underspending in the source industry through the intermediate input cost channel. As a result, underspending reduces output more in the sink industry than in the source industry.

Finally, I present the impulse responses of the model's aggregate variables following a positive monetary shock (figure 3.13). The price of the final good increases immediately after the positive monetary shock (inflation effect). The inflation of the final good price decreases real wages and causes the labour supply to drop lower. The lower labour supply causes lower out-

[11] In the case of two industries ($n = 2$), the star network coincides with the vertical network.

put, consumption and investment. The increase in the money stock is directed to improve the loan supply. This assumption regarding the use of new money means that aggregate financing increases following the positive monetary shock. Following the higher financing levels, it takes one time period for spending to increase, thus increasing real wages and labour supply. The delayed higher labour supply increase causes a delayed positive reaction of output, consumption and investment (liquidity effect). While the surprise positive monetary shock can cause a surprise drop in nominal output, the unexpected drop is too low to cause firms' underspending. This implies that the reaction of the aggregate variables in the main model is identical to the reaction in the benchmark model.

Figure 3.4: Aggregate variables' impulse response functions to a negative productivity shock in industry 1 ($-4 \times \sqrt{2}$ standard deviation) in the main calibration. All variables' responses are presented in a logarithmic form and as a deviation from the steady-state.

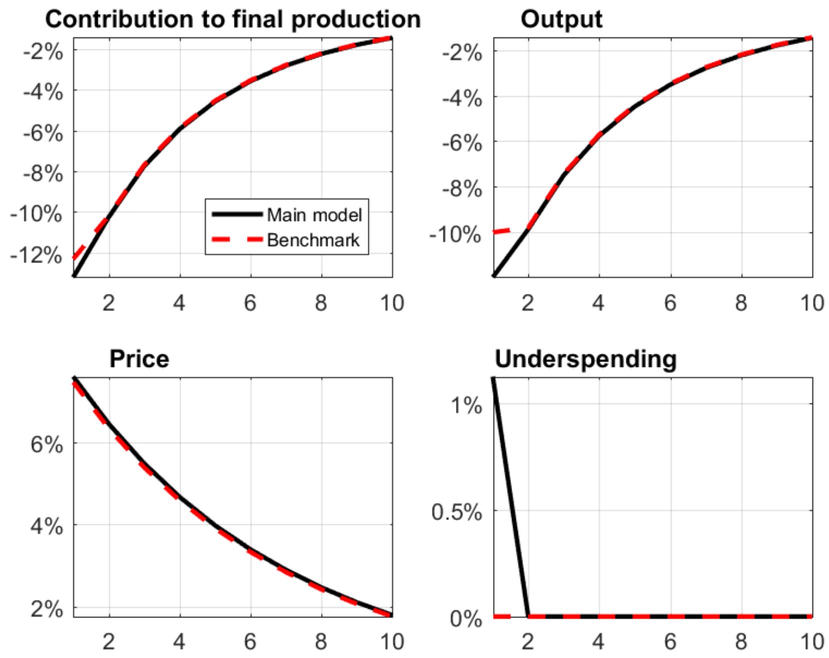

Figure 3.5: Industry 1 variables' impulse response functions to a negative productivity shock in the same industry ($-4\sqrt{2}\times$ standard deviation), assuming the main calibration. All variables' responses are presented in a logarithmic form, and as a deviation from the steady-state, with the exception of underspending that is presented without any transformation.

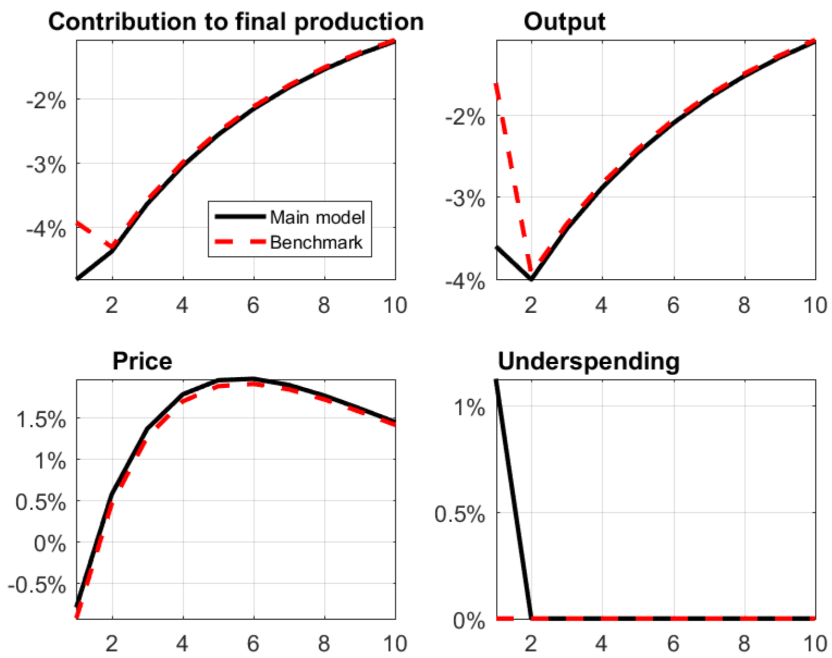

Figure 3.6: Industry 2 variables' impulse response functions to a negative productivity shock in the industry 1 ($-4\sqrt{2}\times$ standard deviation), assuming the main calibration. All variables' responses are presented in a logarithmic form, and as a deviation from the steady-state, with the exception of underspending that is presented without any transformation.

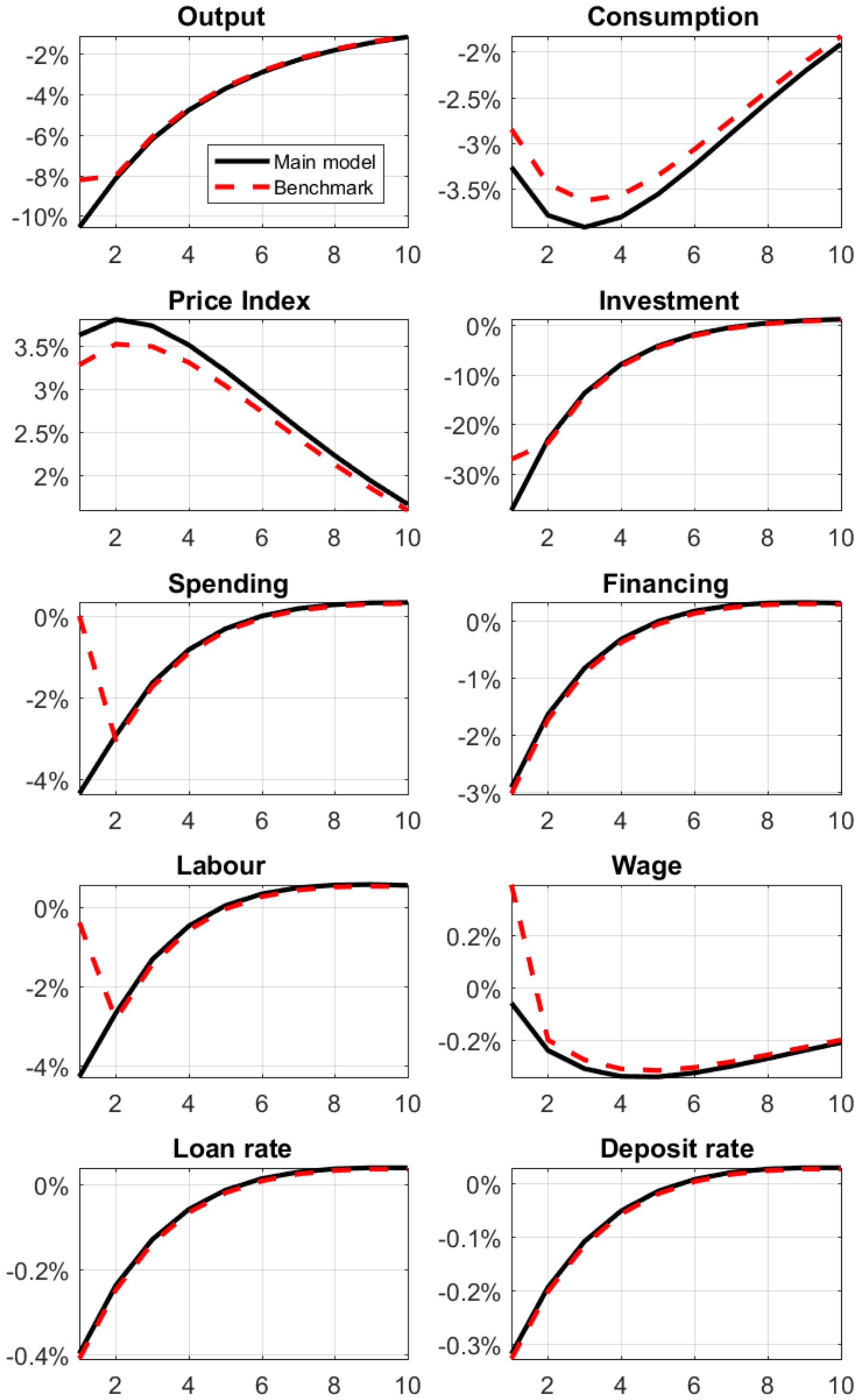

Figure 3.7: Aggregate variables' impulse response functions to a negative productivity shock in industry 1 ($-4 \times \sqrt{2}$ standard deviation), assuming higher Frisch elasticity of labour ($\eta = 0.1$). All variables' responses are presented in a logarithmic form and as a deviation from the steady-state.

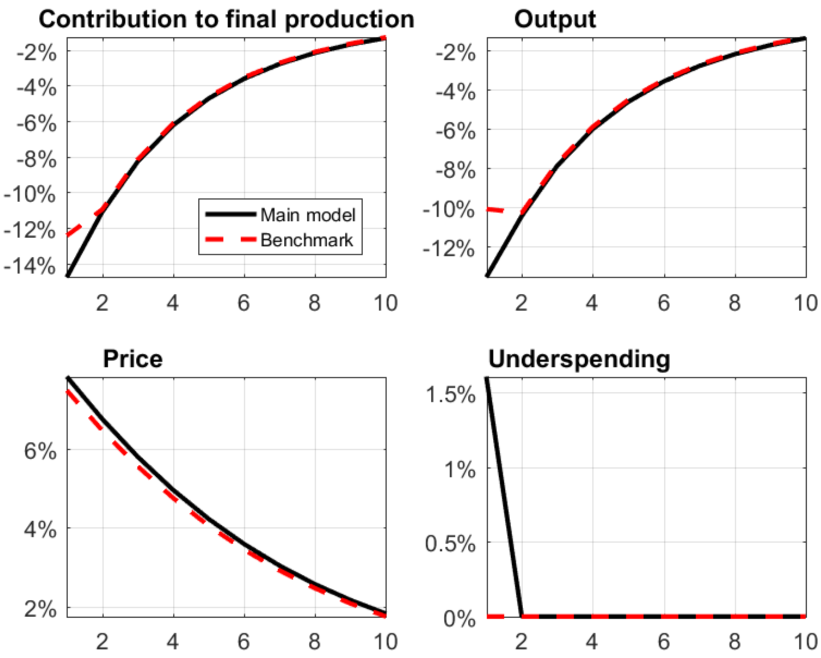

Figure 3.8: Industry 1 variables' impulse response functions to a negative productivity shock in the same industry ($-4\sqrt{2}\times$ standard deviation), assuming higher Frisch elasticity of labour ($\eta = 0.1$). All variables' responses are presented in a logarithmic form, and as a deviation from the steady-state, with the exception of underspending that is presented without any transformation.

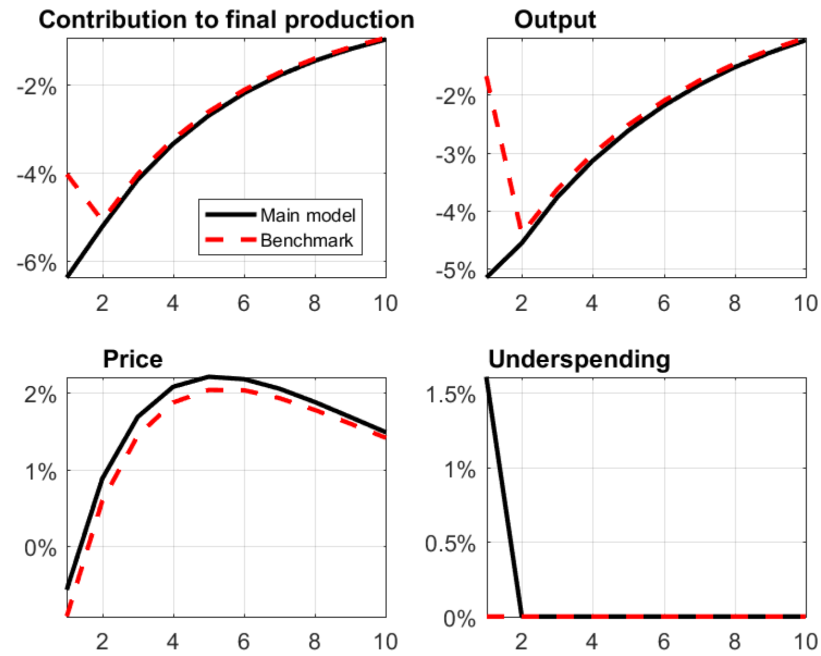

Figure 3.9: Industry 2 variables' impulse response functions to a negative productivity shock in industry 1 ($-4\sqrt{2}\times$ standard deviation), assuming higher Frisch elasticity of labour ($\eta = 0.1$). All variables' responses are presented in a logarithmic form, and as a deviation from the steady-state, with the exception of underspending that is presented without any transformation.

Figure 3.10: Aggregate variables' impulse response functions to a negative productivity shock in industry 1 ($-4.24\times$ standard deviation), assuming a star network. All variables' responses are presented in a logarithmic form, and as a deviation from the steady-state.

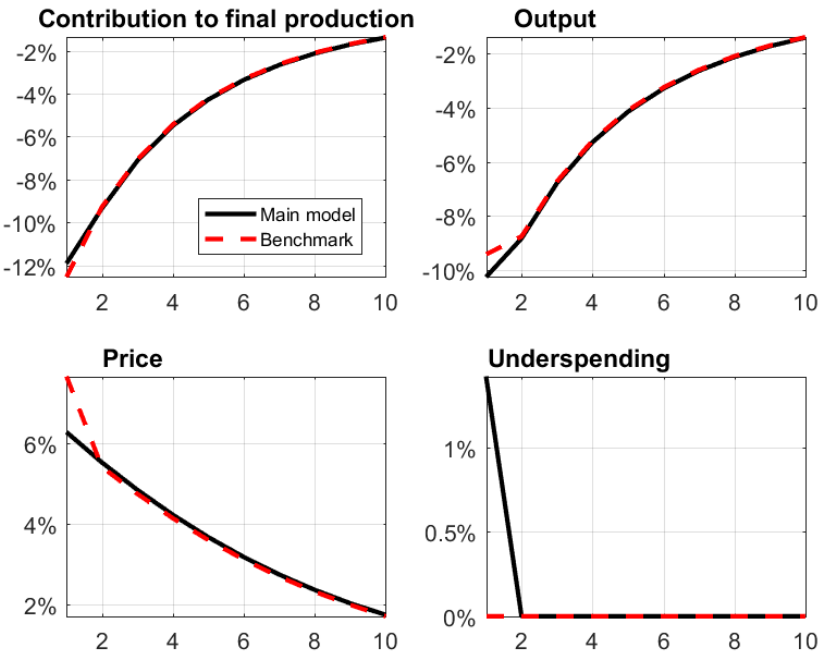

Figure 3.11: Industry 1 variables' impulse response functions to a negative productivity shock in the same industry ($-4.24\times$ standard deviation), assuming a star network. All variables' responses are presented in a logarithmic form, and as a deviation from the steady-state, with the exception of underspending, that is presented without any transformation.

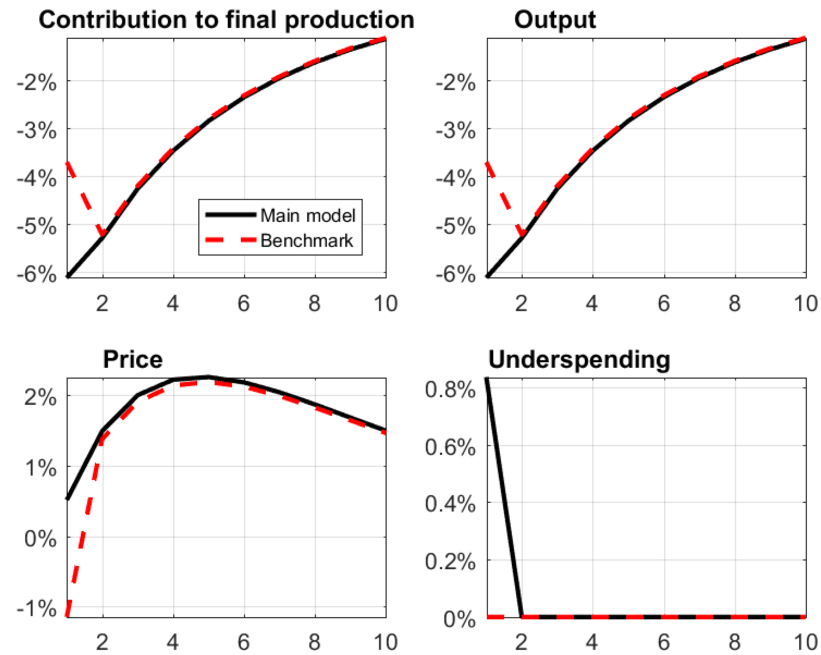

Figure 3.12: Industry 2 variables' impulse response functions to a negative productivity shock in industry 1 ($-4.24\times$ standard deviation), assuming a star network. All variables' responses are presented in a logarithmic form, and as a deviation from the steady-state, with the exception of underspending, that is presented without any transformation.

Figure 3.13: Impulse response functions aggregate variables in the main and benchmark models following a positive shock (1 standard deviation) to the process x^M driving money supply, assuming the main calibration. All variables' responses are presented in a logarithmic form and as a deviation from the steady-state.

3.6 Concluding remarks

I consider firm's underspending in a multi-industry setup where firms are linked through input-output relationships and where idiosyncratic industry-specific shocks drive aggregate fluctuations. I show that a negative productivity shock originating in a subset of industries can propagate through intermediary input channels and make other industries enter a low spending mode where they spend less than the cash previously earmarked for production. These intermediary input channels are key to the functioning of the multi-industry version of the model and provide a potential additional explanation to how crises propagate across industries.

The model's mechanism does not necessarily become irrelevant as the number of industries in the economy grows large. On the contrary, I show that under some assumptions on the nature of the input-output relationships, the likelihood of the mechanism operating remains stable even when the production sector includes a large number of industries. The nature of the input-output production network is key to the aggregation of the model's mechanism: a production network where a small number of industries provide most of the intermediary input in the economy will maintain the model's mechanism irrespective of the degree of industrial diversification. This is in line with the literature on the network origins of aggregate fluctuations.

3.A Technical appendix: multiple industries' model

3.A.1 Multiple industries model theoretical results: proofs

Assume that the first m industries have non binding investment constraints, while the investment constraint binds in the remaining $n-m$ industries. Then, using equation 3.2.31, the invested capital can be written as:

$$\zeta_{i,t}^{1-\alpha} = \alpha e^{u_{i,t}} \frac{p_{i,t}}{\kappa_{i,t}} \text{ for } i = 1, ..., m \tag{3.A.1}$$

$$\zeta_{i,t} = \frac{\iota_{i,t-1}}{1 + x_{t-1}^{M}} \text{ for } i = m+1, ..., n. \tag{3.A.2}$$

Multiply the clearing equation of good i by the price of the same good $p_{i,t}$

$$p_{i,t} x_{i,t} = \sum_{j=1}^{n} p_{i,t} x_{j,i,t} + p_{i,t} y_{i,t}. \tag{3.A.3}$$

Replace using the intermediate producers' and the final producer first order condition for intermediate inputs yields the equation of lemma 1

$$p_{i,t} x_{i,t} = \frac{\alpha_I}{\alpha} \sum_{j=1}^{n} w_{j,i} \zeta_{j,t} + y_t^N \theta_i, \tag{3.A.4}$$

where $y_t^N := p_t y_t$ denotes the nominal output in the modelled economy. The technology function combined with the intermediate production sector first order conditions yield

$$p_{i,t} x_{i,t} = e^{u_{i,t}} \frac{p_{i,t}}{\kappa_{i,t}} \zeta_{i,t}^{\alpha} \tag{3.A.5}$$

Combining 3.A.1 and 3.A.5 yields a linear relationship between firm revenues and expenditure for industries that are not constrained by their financing levels

$$p_{i,t} x_{i,t} = \frac{1}{\alpha} \zeta_{i,t} \text{ for } i = 1, ..., m \tag{3.A.6}$$

Now combine 3.A.6 with 3.A.4 to get an equation implying the levels of invested capital in the industries where the investment constraints are not binding ($i \leq m$)

$$\zeta_{i,t} - \sum_{j=1}^{m} \alpha_I w_{ji} \zeta_{j,t} = \alpha y_t^N \theta_i + \frac{\alpha_I}{1 + x_{t-1}^M} \sum_{j=m+1}^{n} w_{ji} \iota_{j,t-1} \tag{3.A.7}$$

Equations 3.A.7 can be inverted for $Z_{1:m,t} = (\zeta_{1,t}, ..., \zeta_{m,t})'$ as follows

$$Z_{1:m} = \mathscr{L}_m \left\{ \frac{\alpha_I}{1 + x_{t-1}^M} W'_{1:m,m+1:n} I_{m+1:n,t-1} + \alpha y_t^N \Theta_{1:m} \right\} \tag{3.A.8}$$

where, if $X := (x_{i,j})$, we define $X_{p:q,r:s} := (x_{i,j})_{p \leq i \leq q, r \leq j \leq s}$, $\mathscr{L}_m := \left\{ \mathbb{I}_{m,m} - \alpha_I W'_{1:m,1:m} \right\}^{-1}$ and the remaining matrix notations are as above. At this stage, one needs to justify the matrix inversion in the above formula. The matrix $\alpha_I W'_{1:m,1:m}$ is non-negative and the sum of its columns is lower than one, the Perron-Frobenius theorem guarantees that all its left eigenvalues are less than 1 in modulus. This implies that $I_{m,m} - \alpha W'_{1:m,1:m}$ is invertible. The inverse of the latter matrix is also non-negative. This follows from

$$\mathscr{L}_m = \sum_{i=0}^{\infty} (\alpha_I W'_{1:m,1:m})^n. \tag{3.A.9}$$

This completes the proof of proposition 1. Proposition 2 follows immediately from proposition 1 and the model's first order conditions.

Once the expenditure within each industry is determined, one can use equation 3.A.4 to determine the industries revenues $rev_{i,t} := p_{i,t} x_{i,t}$

$$REV_t = \frac{\alpha_I}{\alpha} W' Z_t + y^N(t) \Theta \tag{3.A.10}$$

and then use equation 3.A.5 to determine the ratios $\frac{p_{i,t}}{\kappa_{i,t}}$ in a logarithmic form

$$\tilde{p}_{i,t} - \tilde{\kappa}_{i,t} = r\tilde{e}v_{i,t} - u_{i,t} - \alpha \tilde{\zeta}_{i,t} \tag{3.A.11}$$

Using the the expression for the production costs κ_i, one can write the log-linear system above as follows

$$(\mathbb{I}_{n,n} - \alpha_I W) \tilde{P}(t) = A + \left(\alpha_K \ln(\frac{\alpha}{\alpha_K} p_t r_t^K) + \alpha_L \ln(\frac{\alpha}{\alpha_L} \omega_t) \right) \mathbf{1}_{n,1} + R\tilde{E}V(t) - \alpha \tilde{Z}(t) - U(t), \tag{3.A.12}$$

where $A := (a_i)$ is a constant vector and $a_i := \alpha_I \tilde{\alpha} - \alpha_I \tilde{\alpha}_I - \alpha_I \sum_{j=1}^{n} w_{ij} \tilde{w}_{ij}$. Replace using the capital and labour first order conditions from the intermediate producer spending problem

$$(\mathbb{I}_{n,n} - \alpha_I W)\tilde{P}(t) = A + \left[(\alpha_K + \alpha_L)\tilde{\zeta}_t - \alpha_K \tilde{k}_{t-1} - \alpha_L \tilde{l}_t\right] \mathbf{1}_{n,1} + R\tilde{E}V(t) - \alpha \tilde{Z}(t) - U(t). \quad (3.A.13)$$

This is the first result of proposition 3. Noting that $\tilde{p}_t = \Theta' \tilde{P}_t$, one can write the final good price

$$\tilde{p}_t = (\mathscr{L}\Theta)' A + \frac{1}{1-\alpha_I} \left\{(\alpha_K + \alpha_L)\tilde{\zeta}_t - \alpha_K \tilde{k}_{t-1} - \alpha_L \tilde{l}_t\right\} + (\mathscr{L}\Theta)' \left[R\tilde{E}V_t - \alpha \tilde{Z}_t\right] + u_t, \quad (3.A.14)$$

where $u_t := (\mathscr{L}\Theta)' U_t$. This concludes the proof for all the expressions in proposition 3.

3.A.2 Stead state

The following proposition provides several equations determining the model equilibrium in the steady-state.

Proposition 7. *Under the model closing assumption 2, the steady-state equilibrium of the model exists, is unique and the steady-state variables are determined by the following system of equations*

- *The steady state deposit rate and capital cost are given by*

$$\bar{r} = \frac{1 + x^M}{\beta}, \quad (3.A.15)$$

$$\bar{r}^K = 1/\beta + \delta - 1. \quad (3.A.16)$$

- *The overall financing ι is determined as a function of the steady state deposit rate and the steady-state nominal output*

$$\bar{\iota} = \frac{\alpha(1 + x^M)\bar{y}^N + \bar{r} x^M}{\bar{r} - \alpha_I}. \quad (3.A.17)$$

- *The financing rate is derived from the overall spending*

$$\bar{r}^F = \bar{r}(1 - x^M/\bar{\iota}). \quad (3.A.18)$$

- *The aggregate* $\bar{\zeta} := \bar{\zeta}_1 + ... + \bar{\zeta}_n$ *spending is given by*

$$\bar{\zeta} = \bar{\iota}/(1 + x^M) \tag{3.A.19}$$

- *The steady state consumption spending is*

$$\bar{c}^N = \bar{y}^N - \delta \frac{\alpha_K}{\alpha} \frac{\bar{\zeta}}{\bar{r}^K}. \tag{3.A.20}$$

- *The steady-state labour*

$$\bar{l}^{\eta+1} = \frac{\alpha_L}{\chi\alpha} \frac{\bar{\zeta}}{\bar{c}^N}. \tag{3.A.21}$$

- *The households' deposits are derived from the overall firms' financing*

$$\bar{b} = \bar{\iota} - x^M. \tag{3.A.22}$$

- *The industry specific steady-state investments and are an indirect function of steady-state nominal output*

$$\bar{I} = \alpha \frac{1 + x^M}{\bar{r}^F} \bar{y}^N \left(\mathbb{I}_{n,n} - \frac{\alpha_I}{\bar{r}^F} W' \right)^{-1} \Theta, \tag{3.A.23}$$

where $\mathbb{I}_{n,n}$ *is the square identity matrix with* n *rows and* n *columns.*

- *Firms' spending is constrained in the steady-state*

$$\bar{Z} = \frac{1}{1 + x^M} \bar{I}. \tag{3.A.24}$$

- *The steady-state revenues are derived from the the steady-state financing rate and industry financing*

$$R\bar{E}V = \frac{\bar{r}^F}{\alpha(1 + x^M)} \bar{I}. \tag{3.A.25}$$

- *The steady-state final good price is given by*

$$\frac{1 - \alpha_I - \alpha_K}{1 - \alpha_I} \tilde{p} = \frac{1}{1 - \alpha_I} \left\{ \alpha_K (\tilde{\alpha} - \tilde{\alpha}_K) + \frac{\eta}{1 + \eta} \alpha_L \tilde{\zeta} + \alpha_K \tilde{r}^K + \frac{\alpha_L}{1 + \eta} \tilde{c}_t^N \right\} + (\mathscr{L}\Theta)' \left[\hat{A} + R\tilde{E}V - \alpha \tilde{Z} \right]. \tag{3.A.26}$$

- *the steady-state capital is*

$$\bar{k} = \frac{\alpha_K}{\alpha} \frac{\bar{\zeta}}{\bar{p}\bar{r}^K}. \tag{3.A.27}$$

- *The logarithm of steady-state good prices are expressed as a function of steady-state nominal output and expenditure as follows*

$$\tilde{P} = \mathscr{L}' \left\{ A + \left[(\alpha_K + \alpha_L)\tilde{\zeta} - \alpha_K \tilde{k} - \alpha_L \tilde{l} \right] \mathbf{1}_{n,1} + R\tilde{E}V - \alpha \tilde{Z} \right\}, \tag{3.A.28}$$

where the superscript $\tilde{.}$ *is used for logarithmic values and* A *is defined as above.*

- *steady-state industry inputs are*

$$\bar{x}_{ij} = \frac{\alpha}{\alpha + \gamma} \frac{\bar{\zeta}_i}{\bar{p}_j}. \tag{3.A.29}$$

- *the steady-state final use of each good*

$$\bar{y}_i = r\bar{e}v_i / \bar{p}_i - \sum_{j=1}^{n} x_{ji}. \tag{3.A.30}$$

- *The real money holding by households is*

$$\bar{m}^H = (\bar{p})^{-1/\nu} \left(\psi \frac{\bar{r}}{\bar{r} - 1} \bar{c}^N \right)^{1/\nu}. \tag{3.A.31}$$

- *finally, the steady-state nominal output* $\bar{y}^N$ *is solved for as a function of the model parameters using the equation*

$$\bar{p}^{1-1/\nu} \left(\psi \frac{\bar{r}}{\bar{r} - 1} \bar{c}^N \right)^{1/\nu} + \bar{b} = 1. \tag{3.A.32}$$

Proof. The steady-state version of the Euler equation yields

$$\bar{r} = 1/\beta. \tag{3.A.33}$$

In the absence of unforeseen negative productivity shocks, the financing constraints hold in the steady state: $\bar{\zeta}_i = \bar{\iota}_i / (1 + x^M)$. Under assumption 2 one can rewrite the investment equations in the steady-state as follows

$$\bar{r}^F \bar{\iota}_i = \alpha(1 + x^M) \bar{p}_i \bar{x}_i, \tag{3.A.34}$$

Using lemma 1 the investment equation 3.A.34 yields

$$\frac{\bar{r}^F}{1+x^M}\bar{\iota}_i = \alpha_I \sum_{j=1}^{n} w_{ji}\bar{\zeta}_j + \alpha \bar{y}^N \theta_i. \tag{3.A.35}$$

Alternatively, in matrix format, one can write

$$\left(\mathbb{I}_{n,n} - \frac{\alpha_I}{\bar{r}^F} W'\right) \bar{I} = \alpha \frac{1+x^M}{\bar{r}^F} \bar{y}^N \Theta, \tag{3.A.36}$$

Because $\frac{\alpha_I}{\bar{r}^F}W'$ is a non-negative matrix with column sums less than one, the Perron-Frobenius theorem guarantees that all its left eigenvalues are less than 1 in modulus. This implies that $\mathbb{I}_{n,n} - \frac{\alpha_I}{\bar{r}^F}W'$ is invertible so that one can solve for the steady-state investment levels as follows

$$\bar{I} = \alpha \frac{1+x^M}{\bar{r}^F} \bar{y}^N \left(\mathbb{I}_{n,n} - \frac{\alpha_I}{\bar{r}^F} W'\right)^{-1} \Theta \tag{3.A.37}$$

Note here that $\theta \geq 0$ and $(\mathbb{I}_{n,n} - (\alpha_I/\bar{r}^F)W')^{-1} = \sum_{k=0}^{\infty}((\alpha_I/\bar{r}^F)W')^k > 0$ guarantee that $\bar{I} \geq 0$. More precisely, the industry financing levels ι_i are non-negative, and as long as $\theta_i > 0$ or there exist j and $q \in \mathbb{N}^*$ such as $\theta_j > 0$ and the (i,j) element of the matrix W^q is non nil, then $\bar{\zeta}_i = \bar{\iota}_i/(1+x^M) > 0$. In other terms, a portion of the available financing is dedicated to an industry as long as its produce is directly used in final production or if its produce is used directly or indirectly by another industry which produce is used in final production. Notice that the columns of the matrix W' and the elements of the vector Θ sum to one, which implies that

$$\mathbf{1}_{1,n}.\left(\mathbb{I}_{n,n} - \frac{\alpha_I}{\bar{r}^F} W'\right)^{-1} \Theta = \frac{1}{1-\alpha_I/r^F}. \tag{3.A.38}$$

The overall spending can then be deduced from the vector formula for $\bar{I}$

$$\bar{r}^F \bar{\iota} - \alpha_I \bar{\iota} = \alpha(1+x^M)\bar{y}^N. \tag{3.A.39}$$

The financing rate is derived from the overall spending by combining the banks' zero-profit ($\bar{r}^F\bar{\iota} = \bar{r}\bar{b}$) condition and the clearing of the loan market condition ($\bar{b} = \bar{\iota} - x^M$)

$$\bar{r}^F \bar{\iota} = \bar{r}(\bar{\iota} - x^M). \tag{3.A.40}$$

Combine the later equation with 3.A.39 to find

$$\bar{\iota} = \frac{\alpha(1 + x^M)\bar{y}^N + \bar{r}x^M}{\bar{r} - \alpha_I}. \tag{3.A.41}$$

Combine the capital first order condition $\bar{p}\bar{r}^K\bar{k} = \frac{\alpha_K}{\alpha}\bar{\zeta}$ with the final good clearing condition $\bar{c}^N = \bar{y}^N - \bar{\delta}\bar{p}\bar{k}$ to obtain an expression for consumption spending

$$\bar{c}^N = \bar{y}^N - \bar{\delta}\frac{\alpha_K}{\alpha}\frac{\bar{\zeta}}{\bar{r}^K}. \tag{3.A.42}$$

Write proposition 3 in the steady-state

$$\tilde{p} = \frac{1}{1 - \alpha_I}\left\{\left(\alpha_K + \frac{\eta}{1 + \eta}\alpha_L\right)\tilde{\zeta} - \alpha_K\tilde{k} + \frac{\alpha_L}{1 + \eta}\tilde{c}_t^N\right\} + (\mathscr{L}\Theta)'\left[\hat{A} + R\tilde{E}V - \alpha\tilde{Z}\right], \tag{3.A.43}$$

and replace for $\tilde{k}$ using the intermediate producers capital first order condition $\bar{p}\bar{r}^K\bar{k} = \frac{\alpha_K}{\alpha}\bar{\zeta}$

$$\frac{1 - \alpha_I - \alpha_K}{1 - \alpha_I}\tilde{p} = \frac{1}{1 - \alpha_I}\left\{\alpha_K(\tilde{\alpha} - \tilde{\alpha}_K) + \frac{\eta}{1 + \eta}\alpha_L\tilde{\zeta} + \alpha_K\tilde{r}^K + \frac{\alpha_L}{1 + \eta}\tilde{c}_t^N\right\} + (\mathscr{L}\Theta)'\left[\hat{A} + R\tilde{E}V - \alpha\tilde{Z}\right]. \tag{3.A.44}$$

Use the intermediate production capital first order condition again to deduce capital

$$\bar{k} = \frac{\alpha_K}{\alpha}\frac{\bar{\zeta}}{\bar{p}\bar{r}^K}. \tag{3.A.45}$$

The equation 3.A.31 is derived from the household's first order condition for money holding 3.2.7. The equation 3.A.32, determining the steady-state consumer spending is derived from the money clearing equation. ■

Clearly, the above proposition provides a way to determine all steady-state variables as a function of the model parameters and steady-state nominal output $\bar{y}^N$. In turn, the steady-state nominal output $\bar{y}^N$ is determined by solving equation 3.A.32. The proposition below is useful in establishing the aggregation results of section 3.4.

Proposition 8. *Assume $\nu = 1$. Then all steady-state variables with the exception of the final good price $\bar{p}$ the intermediate good prices $\bar{p}_i$, and the industry spending and financing levels $\bar{\zeta}_i$ and $\bar{\iota}_i$ are independent of the number of industries n.*

Proof. Note that when $\nu = 1$, equation 3.A.32 does not involve the price term $\bar{p}$. Besides $\bar{y}^N$, the remaining variables in equation 3.A.32 are expressed in proposition 3.A.2 as a function of $\bar{y}^N$ and a subset of model parameters that excludes the number of industries n. This implies that $\bar{y}^N$ is not impacted by the parameter n. The independence of other steady-state variables excluding $\bar{p}$, $\bar{\iota}_i$ and $\bar{\zeta}_i$ of the number of industries n follows immediately from proposition 3.A.2. ■

3.A.3 The effect of input/output relationships: proofs

Proof of proposition 5

Writing the binding and non binding constraints conditions in the form below yields the result of proposition 5

$$REV_{m+1:n,t} > \frac{1}{\alpha}\frac{1}{1+x^M_{t-1}} I_{m+1:n,t-1}, \tag{3.A.46}$$

$$Z_{1:m,t} \leq \frac{1}{1+x^M_{t-1}} I_{1:m,t-1}. \tag{3.A.47}$$

Writing lemma 1 for the set of constrained industries and using 3.A.8 yields

$$\begin{aligned} REV_{m+1:n,t} &= y^N_t \left(\alpha_I W'_{m+1:n,1:m} \mathscr{L}_m \Theta_{1:m} + \Theta_{m+1:n}\right) \\ &+ \frac{1}{\alpha}\frac{1}{1+x^M_{t-1}} \left(\alpha_I^2 W'_{m+1:n,1:m} \mathscr{L}_m W'_{1:m,m+1:n} + \alpha_I W'_{m+1:n,m+1:n}\right) I_{m+1:n,t-1}. \end{aligned}$$

Condition 3.A.46 can then be rewritten as

$$\begin{aligned} &\alpha(1+x^M_{t-1}) y^N_t \left(\alpha_I W'_{m+1:n,1:m} \mathscr{L}_m \Theta_{1:m} + \Theta_{m+1:n}\right) > \\ &I_{m+1:n,t-1} - \left(\alpha_I^2 W'_{m+1:n,1:m} \mathscr{L}_m W'_{1:m,m+1:n} + \alpha_I W'_{m+1:n,m+1:n}\right) I_{m+1:n,t-1}, \end{aligned} \tag{3.A.48}$$

while condition 3.A.47 can be rewritten using 3.A.8 as

$$\alpha(1+x^M_{t-1}) y^N_t \mathscr{L}_m \Theta \leq I_{1:m,t-1} - \alpha_I \mathscr{L}_m W'_{1:m,m+1:n} I_{m+1:n,t-1}. \tag{3.A.49}$$

Proof of proposition 6

In the case where the spending constraints are not binding, rewrite equation 3.A.8 for $m = n$:

$$Z(t) = \alpha y_t^N T \tag{3.A.50}$$

where as defined above $T := \mathscr{L}\Theta$. All industry constraints would be non binding if $Z(t) \leq I_{t-1}/(1 + x_{t-1}^M)$, where the inequality is meant in the element by element sense. This is equivalent to:

$$y_t^N \leq \frac{1}{\alpha(1 + x_{t-1}^M)} \min_{\tau_i \neq 0} \frac{\iota_{i,t-1}}{\tau_i} \tag{3.A.51}$$

where the constants τ_i are defined as $(\tau_1, ..., \tau_n)' := T$. These constants are non-negative because the Leontief matrix is positive and the vector Θ is non negative. This proves the first characterisation of proposition 6.

From the financing equation in proposition 4

$$I_{t-1} = \frac{\alpha}{r_{t-1}^F}(1 + x_{t-1}^M)\mathbb{E}_{t-1}[y_t^{N,C}]\Gamma_{t-1}\Theta, \tag{3.A.52}$$

where by definition $\Gamma_{t-1} := \left(\mathbb{I}_{n,n} - \frac{\alpha_I}{\bar{r}_{t-1}^F} W'\right)^{-1}$. One can then rewrite the inequality 3.A.51 as follows:

$$\frac{y_t^N}{\mathbb{E}_{t-1}[y_t^{N,C}]} \leq \frac{1}{\bar{r}_{t-1}^F} \min_{\tau_i \neq 0} \frac{\gamma_{i,t-1}}{\tau_i}, \tag{3.A.53}$$

where $(\gamma_{1,t-1}, ..., \gamma_{n,t-1})' := \Gamma_{t-1}\Theta$. This is the second characterisation of proposition 6.

3.B Numerical procedures

The results of section 3.3 can be used to provide a simulation procedure as detailed the algorithm 1. The algorithm requires linearisation of the model when the spending constraints of all industries are binding. This is realised using Dynare with a second order approximation, taking the model equations in 3.B.1. Note how the model equations imply that the aggregate variables are driven by the fluctuations of aggregate log-productivity $u_t = \sum_{i=1}^n \tau_i u_{i,t}$ and does not depend on the distribution of productivity shocks across industries.

3.B.1 Constrained spending model equations

$$\zeta_t = \iota_{t-1}/(1 + x^M_{t-1}), \tag{3.B.1}$$

$$p_t r^K_t k_t = \frac{\alpha_K}{\alpha}\zeta_t, \tag{3.B.2}$$

$$\omega_t l_t = \frac{\alpha_L}{\alpha}\zeta_t, \tag{3.B.3}$$

$$\chi l^\eta = \frac{\omega_t}{p_t c_t}, \tag{3.B.4}$$

$$\chi l^\eta = \frac{\omega_t}{p_t c_t}, \tag{3.B.5}$$

$$\frac{1 + x^M_t}{p_t c_t} = \beta r_t \mathbf{E}_t \frac{1}{p_{t+1}c_{t+1}}, \tag{3.B.6}$$

$$\frac{1}{c_t} = \beta \mathbf{E}_t \frac{1}{c_{t+1}}(1 - \delta + r^K_{t+1}), \tag{3.B.7}$$

$$\iota_t(r^F_t - \alpha_I) = \alpha(1 + x^M_t)\mathbf{E}_t p_{t+1} y_{t+1}, \tag{3.B.8}$$

$$r^F_t \iota_t = r_t b_t, \tag{3.B.9}$$

$$k_t + c_t = y_t + (1 - \delta)k_{t-1} \tag{3.B.10}$$

$$(m^H_t)^\nu = \psi c_t \frac{r_t}{r_t - 1}, \tag{3.B.11}$$

$$p_t m^H_t + b_t = 1, \tag{3.B.12}$$

$$u_t = \rho u_{t-1} + \sigma^u e_t, \text{ and } e_t \text{ is i.i.d and } N(0,1). \tag{3.B.13}$$

Finally, the price p_t is determined by the equation below

$$\tilde{p}_t = \frac{1}{1 - \alpha_I}\left\{\left(\alpha_K + \frac{\eta}{1+\eta}\alpha_L\right)\tilde{\zeta}_t - \alpha_K \tilde{k}_{t-1} + \frac{\alpha_L}{1+\eta}\tilde{c}^N_t\right\} + (\mathscr{L}\Theta)'\left[\hat{A} + R\tilde{E}V_t - \alpha\tilde{Z}_t\right] - u_t, \tag{3.B.14}$$

where in the last equation $Z_t = \frac{(1-\alpha/r^F_{t-1})\iota_{t-1}}{1+x^M_{t-1}}\left(\mathbb{I}_{n,n} - \frac{\alpha_I}{r^F_{t-1}}W'\right)\Theta$ and $REV_t = \frac{\alpha_I}{\alpha}W'Z_t + p_t y_t \Theta$.

3.B.2 Simulation methodology

Algorithm 1. *1. Through linearising a version of the model where the spending constraints are always binding, compute the policy functions giving the next period's nominal output* $y^{N,C}_{t+1}$*, con-*

sumption spending $c^{N,C}_{t+1}$, capital rent costs $r^{K,C}_{t+1}$ and consumption c^{C}_{t+1}

$$y^{N,C}_{t+1} = \mathscr{Y}^N(\iota_t, r^F_t, k_t, u_t, x^M_t; e_{t+1}, e^M_{t+1}), \tag{3.B.15}$$

$$c^{N,C}_{t+1} = \mathscr{C}^N(\iota_t, r^F_t, k_t, u_t, x^M_t; e_{t+1}, e^M_{t+1}), \tag{3.B.16}$$

$$r^{K,C}_{t+1} = \mathscr{R}^K(\iota_t, r^F_t, k_t, u_t, x^M_t; e_{t+1}, e^M_{t+1}), \tag{3.B.17}$$

$$c^{C}_{t+1} = \mathscr{C}(\iota_t, r^F_t, k_t, u_t, x^M_t; e_{t+1}, e^M_{t+1}). \tag{3.B.18}$$

2. Given ι_{t-1} and r^F_{t-1}, compute previous industry financing levels $\zeta_{i,t}$ using equations 3.3.16

$$\iota_{i,t-1} = \gamma_{i,t-1}\left(1 - \frac{\alpha}{r^F_{t-1}}\right)\iota_{t-1}, \tag{3.B.19}$$

$$(\gamma_{1,t-1}, ..., \gamma_{n,t-1})' = \left(\mathbb{I}_{n,n} - \frac{\alpha}{r^F_{t-1}}W'\right)^{-1}\Theta. \tag{3.B.20}$$

3. Assume a certain subset of industries where spending is not constrained, that we note $i = 1, ..., m$.

4. Assume some values for y^N_t and c^N_t respectively.

5. Given the assumed value of y^N_t compute the industries' spending using 3.3.2 then the industry revenues $rev_{i,t}$ using lemma 1.

$$Z_{1:m,t} = \mathscr{L}_m\left\{\frac{\alpha_I}{1 + x^M_{t-1}}W'_{1:m,m+1:n}I_{m+1:n,t-1} + \alpha y^N_t\Theta_{1:m}\right\}, \tag{3.B.21}$$

$$Z_{m+1:n,t} = I_{m+1:n,t-1}/(1 + x^M_{t-1}), \tag{3.B.22}$$

$$REV_t = \frac{\alpha_I}{\alpha}W'Z_t + y^N_t\Theta. \tag{3.B.23}$$

6. Given the calculated industry revenues REV_t and spendings Z_t, and the assumed consumption spending c^N_t, use equation 3.3.10 to determine the value of the final good price p_t

$$\tilde{p}_t = \frac{1}{1-\alpha_I}\left\{\left(\alpha_K + \frac{\eta}{1+\eta}\alpha_L\right)\tilde{\zeta}_t - \alpha_K\tilde{k}_{t-1} + \frac{\alpha_L}{1+\eta}\tilde{c}^N_t\right\} + (\mathscr{L}\Theta)'\left[\hat{A} + R\tilde{E}V_t - \alpha\tilde{Z}_t\right] - u_t. \tag{3.B.24}$$

7. *Compute aggregate labour using equation 3.3.8*

$$\tilde{l}_t = \frac{1}{\eta+1}(\tilde{\alpha}_L - \tilde{\alpha} - \tilde{\chi} + \tilde{\zeta}_t - \tilde{c_t^N}). \tag{3.B.25}$$

8. *Compute the intermediate goods' prices using equation 3.3.3*

$$\tilde{P}_t = \mathscr{L}' \left\{ A + \left[(\alpha_K + \alpha_L)\tilde{\zeta}_t - \alpha_K \tilde{k}_{t-1} - \alpha_L \tilde{l}_t \right] \mathbf{1}_{n,1} + R\tilde{E}V_t - \alpha \tilde{Z}_t - U_t \right\}. \tag{3.B.26}$$

9. *Given the final good price p_t and the previously accumulated capital k_{t-1} compute the rent on capital r_t^K using the capital first order conditions*

$$r_t^K = \frac{\alpha_K}{\alpha} \frac{\zeta_t}{p_t k_{t-1}}. \tag{3.B.27}$$

10. *Compute the new accumulated capital through the final good clearing condition given the assumed values of y_t^N and c_t^N*

$$k_t = (1-\delta)k_{t-1} + \frac{1}{p_t}(y_t^N - c_t^N). \tag{3.B.28}$$

11. *Solve the system composed of the two equations below to determine the values of aggregate financing ι_t and the financing (gross) cost r_t^F*

$$\iota_t = \frac{\alpha+\gamma}{r_t^F - \alpha}(1 + x_t^M)\mathbb{E}_t[\mathscr{Y}^N(\iota_t, r_t^F, k_t, u_t, x_t^M; e_{t+1}, e_{t+1}^M)], \tag{3.B.29}$$

$$r_t^F \iota_t = \frac{1+x_t^M}{\beta}(\iota_t - x_t^M)\left[c_t^N \mathbf{E}_t [\frac{1}{\mathscr{C}^N(\iota_t, r_t^F, k_t, u_t, x_t^M; e_{t+1}, e_{t+1}^M)}] \right]^{-1}. \tag{3.B.30}$$

The first equation of this system is derived from 3.3.17 by replacing the term y_{t+1}^N using the function $\mathscr{Y}^N$. The second equation is the result of plugging the loans market clearing condition and the household's deposits' Euler equation into the banks' zero-profit condition, and replacing the term c_{t+1}^N using the policy function $\mathscr{C}^N$.

12. *Compute industry specific financing levels using the calculated values of ι_t and r_t^F*

$$\iota_{i,t} = \gamma_{i,t}(1 - \frac{\alpha}{r_t^F})\iota_t, \tag{3.B.31}$$

with $(\gamma_{1,t},...,\gamma_{n,t})' = \left(\mathbb{I}_{n,n} - \frac{\alpha}{r_t^F}W'\right)^{-1}\Theta$.

13. *Given the values of* ι_t *and* r_t^F*, compute deposits* b_t *and the deposit rate* r_t

$$b_t = \iota_t - x_t^M, \tag{3.B.32}$$

$$r_t = r_t^F \iota_t / b_t. \tag{3.B.33}$$

14. *Given the calculated value of* r_t*,* p_t *and* c_t^N*, calculate the real consumption* c_t *and the households' money holding* m_t^H

$$c_t = c_t^N / p_t, \tag{3.B.34}$$

$$m_t^H = \left(\psi \frac{r_t}{r_t - 1} c_t\right)^{1/\nu}. \tag{3.B.35}$$

15. *Compute the following error term based on the money clearing equation and the capital's Euler first order condition*

$$Err1 = p_t m^H + b_t - 1. \tag{3.B.36}$$

$$Err2 = \beta c_t \mathbf{E}_t \frac{1}{\mathscr{C}(\iota_t, r_t^F, k_t, u_{t+1}, x_{t+1}^M)} \left\{1 - \delta + \mathscr{R}^K(\iota_t, r_t^F, k_t, u_{t+1}, x_{t+1}^M)\right\} - 1. \tag{3.B.37}$$

16. *Vary the values of* y_t^N *and* c_t^N *so that the error terms* $Err1$ *and* $Err2$ *are small enough.*

17. *Check that the marginal return on spending is smaller than one for industries* $i = 1, ..., m$ *and greater than one for industries* $i = m + 1, ..., n$.

18. *Vary the assumed set of unconstrained industries till the marginal return on spending conditions in the previous step are verified.*

3.B.3 Numerical tests

The main approximation in algorithm 1 concerns the use of perturbation methods to obtain policy function used to model the behaviour of several next period variables. To test this approximation, I study the impulse responses of a version of the model where spending is always constrained, assuming the main calibration described in section 3.5. The variables studied are the ones for which approximation policy functions are obtained through Dynare. These are nominal output, nominal consumption, capital costs and (real) consumption. I compare the results obtained using the exact solutions for today's variables provided in algorithm 1 to those emanating from the policy functions approximations. The size of the aggregate shock corresponds to the one implied by the industry shocks in section 3.5. Given that the steady-state is the same for both models, results are presented in log form and as a deviation from the steady-state. The results are presented in figure 3.14 and show negligible differences for nominal output, nominal consumption and capital costs and a slight difference for consumption (of the order of 0.1%).

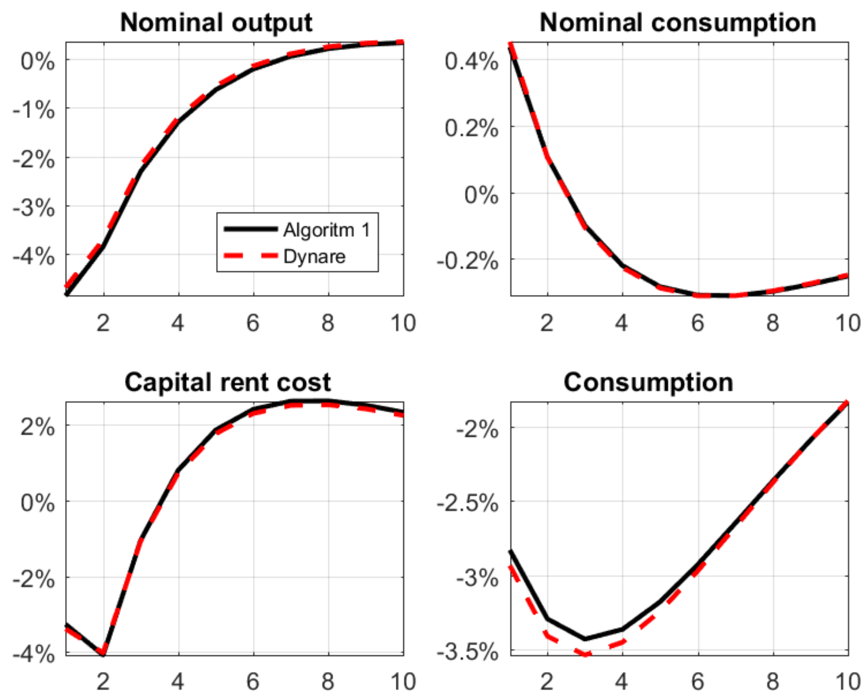

Figure 3.14: Impulse response functions to a negative aggregate productivity shock ($-4\times$ standard deviations) of a version of the model assuming that financing constraints are always binding (main calibration). The continuous lines represent results obtained using the numerical method suggested studied in the appendix, and the dashed lines represent Dynare results. All variables' responses are presented in a logarithmic form and as a deviation from the steady-state.

Chapter 4

Investment Opportunity Indicators and Investors' Rewards

4.1 Introduction

This chapter is an empirical study of the relationship between the firm's investment opportunity indicators and the cash rewards received by investors holding ordinary shares of the firm. I consider three investment opportunity indicators. First, there is the actual investment expense scaled by the firm's previous assets' value. This ratio provides a measure of the firm's current investment activities and is therefore indicative of the firm's inclination to invest and the investment opportunity it faces. Secondly, I consider the market to book ratio, defined as the market capitalisation divided by the firm's previous period assets' value. The higher the firm's market valuation for a given level of assets, the more market participants value the firm's growth potential. Finally, it is natural to assume that more productive firms dispose of more growth opportunities. I, therefore, consider the firm's ability to produce, proxied by Total Factor Productivity (TFP), as a third growth indicator. These growth indicators are used to predict future cash rewards received by ordinary shareholders in the form of dividends and shares buybacks.

The results of this chapter can be used as an empirical foundation for the main mechanism of the models studied in chapters 2 and 3. This mechanism assumes that firms finance production before using the funds raised to pay for production costs. If the firm's productivity unexpectedly drops between the financing stage and the spending stage, the firm's management can choose to cancel some of its spending plans made at the financing stage and return a portion of the cash raised to investors without investing it in the production process. To provide an empirical foundation for such a mechanism, one would ideally need a way to measure changes in the spending intentions of firms, associate these changes of intention with an increase of the cash diverted towards shareholders and link this process to a worsening in the firm's productivity. The data available to me does not provide a way to measure such changes in the firm's spending intentions. In place of measuring changes in the firms' spending intentions, I focus on changes to the cash they divert towards investors. I construct a measure of the overall cash diverted towards shareholders that includes both dividends and share buybacks (”Distributed Cash”). I then document the marginal effect of productivity and other growth indicators on the propensity to divert cash towards shareholders using cross-sectional logit regressions repeated for every year of the studied sample period. I assume that an increase in shareholder payout beyond what can be explained by the relevant firm's characteristics and cash flow figures signals a reduction in the firm appetite towards spending. Thus, an inverse marginal relationship between productivity and ”Distributed Cash” is an indication of a positive marginal relationship between productivity and production spending. To confirm and strengthen the results from the cross-sectional logit regressions, I run a fixed effect panel data regression explaining the size of the payout made by firms choosing to distribute some cash towards shareholders.

I build on the work of Fama and French (2001) who consider the reasons behind the decline in the proportion of public firms paying dividends in the period spanning 1978 to 1999 and the work of Grullon and Michaely (2004) on the information content of share repurchase programs. I also build on the work of Imrohoroglu and Tuzel (2014) that provides evidence regarding the link between firm-level total factor productivity (TFP) and future stock returns. In particular, I use a similar definition of productivity as the one used in Imrohoroglu and Tuzel (2014), the repeated logit regressions in section 4.3.2 are inspired by Fama and French (2001). Furthermore, I follow the methodologies in Grullon and Michaely (2004) and Fama and French (2001) to extract share buybacks from the available data.

I find that firms with low productivity are more likely to divert cash towards shareholders and that, among the set of firms choosing to reward investors, the amount diverted is higher for firms with lower productivity. This is in line with the existing corporate finance literature and provides further confirmation to the Jensen (1986) free cash-flow assumption stipulating that firms tend to increase their cash payouts in response to a deterioration in the set of investment opportunities they face.

Related Literature.— The empirical study in this chapter builds on the existing finance literature concerned with explaining the levels of cash distributed by firms towards shareholders. These cash distributions take two important forms: dividends and share buybacks. Jensen (1986) argues that, when the firm is facing less attractive investment opportunities, a conflict of interest arises between shareholders and managers, with the latter having an incentive to keep more resources under their control and thus not distributing free cash flows. Share repurchases in this context can work as a way to reassure markets about this potential conflict of interests. Grullon and Michaely (2004) find that repurchasing firms reduce their current levels of capital expenditures and research and development expenses and that their cash balances significantly decline. This corroborates the deterioration of the investment opportunities hypothesis. They also find that, contrary to what is suggested by the signalling hypothesis, the markets do not always react positively to the announcement of share repurchases, as market participants are not always aware of the reduction of investments opportunities available to the firm before the share buyback programme is announced. Hribar, Jenkins, and Johnson (2006) show that there is a strong discontinuity in the probability of accretive share repurchases around the consensus earnings per share (EPS) expected by financial analysts. Firms that would have narrowly missed the analysts' consensus EPS are much more likely to increase their share repurchase activity with the goal of positively affecting their EPS and meeting the consensus than those who narrowly beat the consensus EPS. Almeida, Fos, and Kronlund (2016) exploit this discontinuity to show that EPS-motivated share buybacks are associated with reductions in employment and investments. Fama and French (2001) focus on the more usual way chosen by firms to divert cash towards shareholders: dividends. They study the decline in the distribution of dividends by publicly traded firms in the last 20 years of the twentieth century and relate the said decline to many contributing factors, including a change in the characteristics of public firms (firms go public earlier in their development process) and the emergence of competing ways to pay shareholders (mainly share buybacks). The authors also document an empirical inverse relationship between the firms' propensity to pay dividends and the investment opportunity it faces. Since the early 80s, share repurchases make a significant part of the cash flows directed by firms towards investors. I, therefore, construct an index combining both dividends and cash repurchases to account for all cash flows directed towards equity investors as opposed to those being invested in the production processes. This follows the literature concerned with the total cash flow distributed by firms to equity investors. Bagwell and Shoven (1989) give an early account of the increasing roles of share redistribution and take-overs as ways to distribute cash from firms towards equity investors and suggest that yields of return on equity investments should account for these ways of cash distribution. Robertson and Wright (2006) construct a total cash flow index that takes into account dividends, share repurchases and net share issues and use the constructed index to predict stock returns. Imrohoroglu and Tuzel (2014) use total factor productivity (TFP) to predict equity returns and show that while TFP underperforms other indicators such as the market to book ratio in predicting equity returns, low productivity firms earn a significant premium over high productivity firms in the following year. In this paper, I use various firm indicators to explain the propensity of firms to divert cash towards shareholders. Following existing literature, these indicators include investments in capital, research and development and employment. In this regard, my results further validate the idea that firms react to lower investment opportunities by diverting cash towards shareholders. To provide an empirical foundation to the mechanism presented in this paper, I show that besides the investment indicators, total factor productivity helps predict the levels of cash diverted to equity investors. To this effect, I present evidence from repeated cross-sectional logit regressions documenting the propensity of firms to pay shareholders. Additionally, I present dynamic panel data regressions explaining the size of the payout when the firm decides to pay.

4.2 Data

4.2.1 Cash Distributed to Equity Holders

Firms can divert cash towards shareholders in different manners. The method chosen depends, among other things, on the intended recipients, the aim behind the distribution and its tax implications. Namely, firms distribute cash to ordinary equity holders through three important channels:

- **Dividends** are the most common way for a firm to distribute cash to shareholders. They are subject to corporate taxation and to taxes on revenue. Dividends are typically paid in a periodic fashion. This implies that starting to pay dividends or increasing their amount creates an expectation of such behaviour continuing in the future.

- Firms can decide to buy back their own shares (**Share Buybacks**). This can happen through fixed price tender offers and since 1982 mostly through open market operations.[1] After selling all or part of their shares, ordinary equity holders are subject to taxes on capital gains. Capital gains tax rates are typically lower than revenue tax rates, they exclude the cost at which the shares were bought and can be netted against capital losses from other investments by the seller. Share repurchases are therefore at a significant advantage relative to dividends from a tax perspective.

- Firms can also distribute cash towards equity holders through cash financed **mergers and acquisitions**. I will not focus on this particular channel for two reasons. First, when firms buy shares of other companies during a merger and acquisition process, they are typically paying the shareholders of other companies. More importantly, using cash

[1] The 10b-18 rule of 1982 provides guarantees to the firms willing to repurchased their own stock that they would not be in breach of stock manipulation rules if they adhere to certain conditions (Safe Harbor conditions) regarding the manner, timing, price and size of the repurchase. This regulation and others implemented around the same time period simplified the execution of share buybacks and limited the legal liability facing the repurchasing firm.

to finance the acquisition of another company can be considered as a form of investment in the physical, human and intangible capital of the acquired company.

Following the more lenient 1982 regulations, share buybacks have emerged in the mid 1980s as a major way to compensate equity holders beside dividend payments. The left panel of figure 4.1 illustrates this trend and shows the evolution of the average yearly share repurchases versus the annual dividends over time in the sample provided by Compustat for firms based in the United States. The right panel of figure 4.1 also shows that from the early 1970s a significant number of firms chose to buy their own stock back while not distributing dividends. Taking into consideration both dividends and share buybacks is therefore important when studying how firms decide to divert cash towards shareholders. One can hardly interpret a merger and acquision as a signal that the acquiring company is motivated by a need to reduce investment to a lower levels than previously intended.

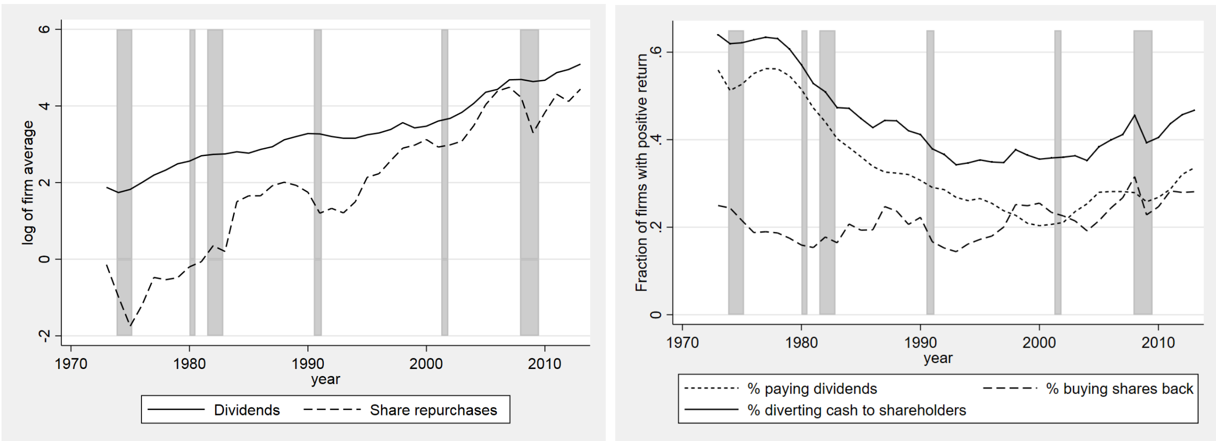

Figure 4.1: Evolution over time of the average amount distributed through dividends and share buybacks in log format by U.S. firms covered by Compustat (left). Proportion of Compustat U.S. firms with positive cash return to equity holders through: dividends only, share buybacks only and a combination of the dividend and share buybacks (right). Grey areas indicate NBER recession periods. Appendix 4.A.1 explains how share buybacks are derived and other data treatments.

4.2.2 Data description

I try to explain three figures representing the cash diverted by the firm to the ordinary shareholders: dividends, share repurchases and "Distributed Cash" defined as the sum of both dividends and share buybacks. The "Distributed Cash" is a measure of the overall cash diverted to common shareholders. The cost to the company is different and depends on the tax treatment of dividends and share buybacks. A number of firm's characteristics and financial indicators are used in order to explain the flow of funds towards ordinary shareholders. Three stand out as reflecting the firm's appetite for growth. These are productivity, investment expenses and the market to book ratio. High investment expenses are a direct indication that the firm is in a growth mode while a high market to book ratio can reflect a view by market participants that the firm has a high potential for growth. Total Factor Productivity (TFP) is used as a proxy for the firm's efficiency of production and is typically high for growth firms.[2] TFP is obtained following Imrohoroglu and Tuzel (2014)

$$TFP = \frac{g}{k^{0.22} \times l^{0.75}}, \tag{4.2.1}$$

where g is value added by the firm, the firm-level capital stock k is given by gross plant, property and equipment (PPEGT) and the stock of labour l is given by the number of employees. In order to remove industry specific TFP effects, the firm level log TFP is corrected by removing 2 digits yearly industry averages. Investment expenses include investment in capital (CAPEX) and in research and development (R&D). The market to book ratio is defined as the market value of ordinary equity divided by the previous period's assets' value.[3]

I control for a number of firm level characteristics and cash-flow figures. These include size related controls, namely, the firm's asset value and market capitalisation. I also control for net income as an indicator of the firm's profitability. High levels of cash and cash equivalent assets may indicate the presence of idle financial resources, providing a motivation for the firm to reward shareholders. I therefore include a measure of cash and cash equivalent assets to the set of control variables. The number of employees is also included as it both serves as a size indicator and an indicator of the firm's wiliness to hire.

I use Compustat US data to get or derive all the firm level variables of interest.[4] Following

[2]See Imrohoroglu and Tuzel (2014).

[3]Another definition of the market to book ratio is the market value of the firm divided by its book value. Adopting this definition restricts the size of the sample significantly as the book value data is only available for a small protion of firms.

[4]The cost of labour data is obtained by multiplying the Compustat number of employees by the average

existing corporate finance literature, utilities and financial industry firms are removed from the sample as these companies are subject to specific regulations that impact dividends' distribution. To avoid outliers, firms with no assets are also excluded. Share repurchases are defined, following Fama and French (2001), as the increase in Treasury stock if the Treasury stock is not missing. Following Almeida, Fos, and Kronlund (2016), if the Treasury stock is missing in the current or prior year, share repurchases are measured as the difference between stock purchases and stock issuances using the cash flow statements. If either measures is negative, share repurchases are set to zero for the corresponding period. This data treatment is maintained for the rest of this chapter.

Many of the studied firm characteristics and financial data vary in magnitude for a single firm throughout the firm's life cycle and between firms of various sizes for a given year. Without any scaling, big firms would influence the regressions' results more than small firms and later periods of the sample would influence the results more than earlier periods due to the combined effects of inflation and capital accumulation. In order to correct for these effects, I scale all of the variables, except for the market to book ratio that does not require scaling and the assets' value that I keep as an unscaled measure of the size of the firm. Debt, cash holdings, net income, CAPEX, R&D investments, the number of employees and TFP are divided by the value of the previous period's assets.[5] The market capitalisation, is replaced by its percentile equivalent.[6] Dividends, share buybacks and the distributed cash are divided by the previous market capitalisation. All variables, except for the market capitalisation percentile, are Winsorised at the 1% level to correct for the outliers' effect. Appendix 4.A.1 provides more details about the definition of the derived variables as well as a summary of the transformation applied to the data.

wages from the Social Security Administration. As explained in appendix 4.A.1, the cost of labour is useful in the derivation of the firm level value added.

[5] These scaling choices are, to a large extent, inspired by Fama and French (2001).

[6] By percentile form of a variable X_t, I mean the transformation $\text{Percentile}_t^X(x) = 100 \times \frac{\text{number of observations satisfying } X_t < x}{\text{number of observations at time } t}$.

4.3 Empirical evidence

4.3.1 Growth indicators and firms' payouts

Table 4.1 presents summary statistics of the distributed cash to market capitalisation ratios for observations sorted using the growth indicators described in section 4.2. For every fiscal year, I calculate the deciles of all growth indicators and divide the firms in every year in groups delimited by two consecutive deciles. I then calculate the average cash returned for each of the defined groups of observations over the period between 1980 and 2013. The results show a tendency for the average payout ratios to be lower for higher deciles of Market to Book, TFP, CAPEX and R&D spending. This relationship is strongest for the Market to Book decile groups where the average cash returned to market capitalisation ratio monotonically decreases from lower to higher deciles. The average cash distributed to market capitalisation in the 90%-100% market to book decile is 0.47%. It is much higher in the 0%-10% decile at 40.9%. A similar pattern is observed when considering TFP decile groups. The average cash distributed to market capitalisation ratio for 0%-10% TFP decile is 31.0% while it stands at 1.34% for the highest TFP decile. The relationship between TFP and the distributed cash to market capitalisation ratio is mostly decreasing, with the monotony being broken only for the 50%-60% and 60%-70% deciles. The lowest decile in terms of CAPEX spending has an average distributed cash to market capitalisation ratio of 4.15% while the highest decile has an average payout ratio of 1.48%. While there is no clear decreasing behaviour of the distributed cash to market capitalisation ratio with relation to CAPEX deciles, the average payout to market capitalisation ratio is smallest for the 90%-100% decile. R&D spending implied deciles display a similar behaviour to the CAPEX implied deciles, with the average distributed cash to market capitalisation being lowest for the 90%-100% decile at 0.26%.

Given that the market capitalisation appears in the nominator of the definition of the market of book ratio and in the denominator of the distributed cash to market capitalisation ratio, the results for the market to book deciles may appear harder to interpret. However, TFP decile groups display a similar behaviour to the market to book deciles' groups while market capitalisation play no role in the definition of TFP. In addition, the strong relationship between the market to book ratio and the distributed cash is confirmed by the results of logit regressions presented in the next subsection.

Percentile	Distributed Cash / Market Cap. - Average			
%	Mkt to Book Decile	TFP Decile	CAPEX Decile	R&D Decile
0-10	0.4090	0.3198	0.0415	0.0253
10-20	0.0336	0.1160	0.0577	0.0721
20-30	0.0309	0.0484	0.0355	0.0512
30-40	0.0287	0.0306	0.0476	0.2224
40-50	0.0256	0.0258	0.0782	0.0497
50-60	0.0231	0.0583	0.0761	0.0256
60-70	0.0194	0.0311	0.0686	0.0718
70-80	0.0158	0.0189	0.0727	0.0140
80-90	0.0110	0.0148	0.1108	0.0084
90-100	0.0047	0.0134	0.0148	0.0026

Table 4.1: Average cash distributed to market capiatlisation ratio by growth indicator decile buckets. Growth Indicator decile cutt-off points are recomputed for every year of the sample period. The growth indicators are defined, scaled and transformed as described in section 4.2.2. Data from 1980 to 2013.

4.3.2 Firm's propensity to pay: evidence from repeated cross-sectional logit regressions

In order to explain the decision of the firm's management to payback investors, I run a series of cross-section logit regressions repeated for every fiscal year between 1980 and 2013, where the dependent variable is the "Distributed Cash" and the explanatory variables of interest are lagged indicators for the firm's appetite to grow: TFP, market to book ratio and the investment expenses (CAPEX and R&D). To avoid competing effects between these growth indicators, separate regressions are run to get the respective marginal effect of TFP, market to book ratio and the combined effects of CAPEX and R&D expenditures. In addition, I run regressions including all the growth indicators to assess their combined effects. The market capitalisation percentile, assets, cash and cash equivalents, net income, debt and number of employees are used as lagged controls in all the regressions. Two digit industry dummies are also included in the regressions to account for industry related effects. The repeated logit regressions can be described by the equation

$$y_t = \beta_{x,t} x_{t-1} + \beta_{z,t} z_{t-1} + \text{industry dummies}, \tag{4.3.1}$$

where y_t is the "distributed cash" variable, x_{t-1} denotes the lagged growth indicators of interest, and z_{t-1} are the lagged controls

$$x_{t-1} := \begin{cases} \text{TFP} \\ \boldsymbol{OR} \\ \text{Market/Book} \\ \boldsymbol{OR} \\ [\text{CAPEX, R\&D}] \\ \boldsymbol{OR} \\ [\text{TFP, Market/Book, CAPEX, R\&D}], \end{cases}$$

$$z_{t-1} = [\text{Market Capitalisation, Assets, Cash, Net Income, Debt, Employees}].$$

Figure 4.2 reports the estimated coefficients $\beta_{x,t}$ of the growth indicators and the corresponding 95% confidence intervals for the logit regression 4.3.1, repeated for every year from 1980 to 2013. Figures 4.4 and 4.5 provide the same coefficients and confidence intervals over the same time period when the regression 4.3.1 is used to explain dividends and share buybacks, respectively.

Firms with high growth indicators are less likely to divert cash towards investors, the effects being both economically and statistically significant.[7] Firms with higher market/book, CAPEX and R&D investments have a lower propensity to reward shareholders. This is consistent with Fama and French (2001) who find that firms with high investment opportunities as reflected by high asset growth rates and high market to book ratios are less likely to pay dividends. The results presented here confirm these findings when including share buybacks in the measure of

[7]See appendix 4.A.2 for means and standard deviations of the growth indicators.

cash diverted towards equity holders. Although TFP has a statistically weaker marginal effect when compared to other growth indicators, its effect remains both statistically and economically significant for most of the studied period.[8] Lower TFP leads to higher propensity to divert cash towards shareholders, thus providing an empirical justification to the model in chapter 2.

When simultaneously including all the growth indicators in the regressions, the effects of the market to book ratio and R&D dominate the effects of other growth indicators (figure 4.3). The marginal effects of CAPEX and TFP remain negative for most of the studied period but lose their statistical significance in most years. This is consistent with Imrohoroglu and Tuzel (2014) who find that firms with low TFP have significantly higher equity returns in the following year but that the TFP effect is not significant when other predictors, such as the market to book ratio, are used alongside TFP to predict equity returns.

The separate marginal effects of the growth indicators on dividends and share buybacks are shown in figures 4.4 and 4.5. The results indicate that firms with lower TFP are more likely to pay dividends and are more likely to buy their own shares back with this effect being more pronounced in the case of dividends payments than in the case of shares buybacks. In addition, the market to book ratio and R&D spending effects are maintained when considering the propensity to pay dividends and repurchase shares separately. Firms with lower market to book ratio are more inclined to pay dividend and buy their shares back and the same applies for firms with low R&D spending. The magnitude of both the market to book effect and the R&D effects is stronger when explaining the propensity to pay dividends.

The marginal effects of the used controls are presented in appendix 4.A.3. The results confirm the literature findings with regard to the relationship between some of the firm's characteristics and the propensity to pay shareholders. Large firms, meaning those with large assets and high market capitalisation tend to distribute more cash towards equity holders. Furthermore, as one may expect, firms that are burdened by relatively high debt levels are less likely to pay equity investors, the debt effect being significant for almost every year of the studied time

[8]TFP underperforms the market to book ratio, possibly because TFP is measured with some noise. For example, the formula 4.2.1 defining TFP assumes the same exponents for labour and capital for all firms over the full sample period.

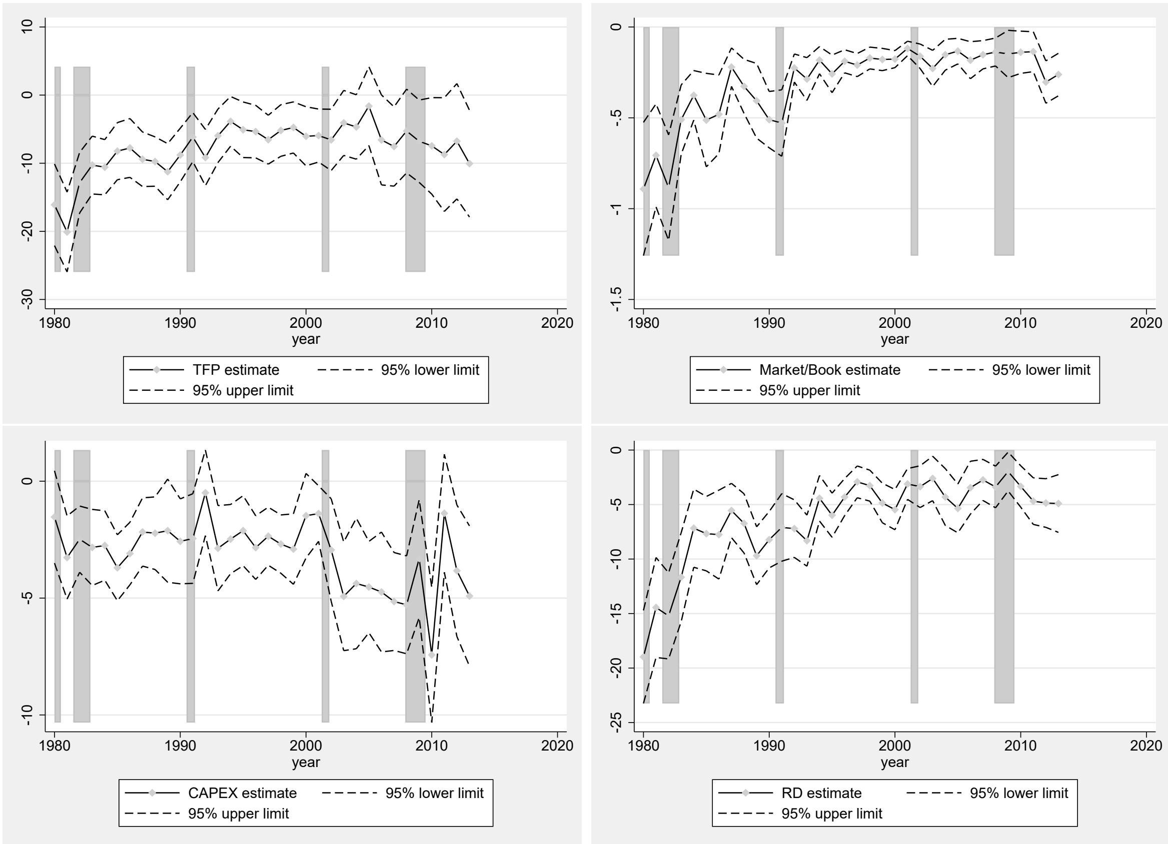

Figure 4.2: Repeated logit cross-section regressions estimates with 95% confidence boundaries corresponding to the effects of TFP, Market/Book and investment variables (CAPEX and R&D) on the firm's propensity to pay shareholders. The logit regressions are repeated for every year from 1980 to 2013. The TFP marginal effect is estimated without controlling for Market/Book, CAPEX and R&D, The Market/Book marginal effect is estimated without controlling for TFP, CAPEX and R&D and the CAPEX and R&D effects are estimated in the same repeated regressions that exclude both TFP and Market/Book. Controls common to all regressions include: market capitalisation, assets, cash, net income, debt and the number of employees. Grey areas indicate NBER recession periods.

period. Lastly, the evidence from the logit regressions shows no significant impact of cash on the propensity to pay. This is not an intuitive result. It is legitimate to suspect that high levels of cash holdings may indicate a low investment opportunity and therefore incite the firm to pay equity holders. I propose two possible justifications for the non intuitive cash effect. First, the static nature of the regressions does not allow for taking into account the firm's idiosyncratic need of holding cash. For example, firms may keep hold of relatively high cash amounts because of a lack of access to capital markets, thus a high cash holding relative to assets might reflect that the firm is still in an early development stage and has not reached the size where it can rely on capital markets to help manage its cash flows. Firms that are still in the early stages of

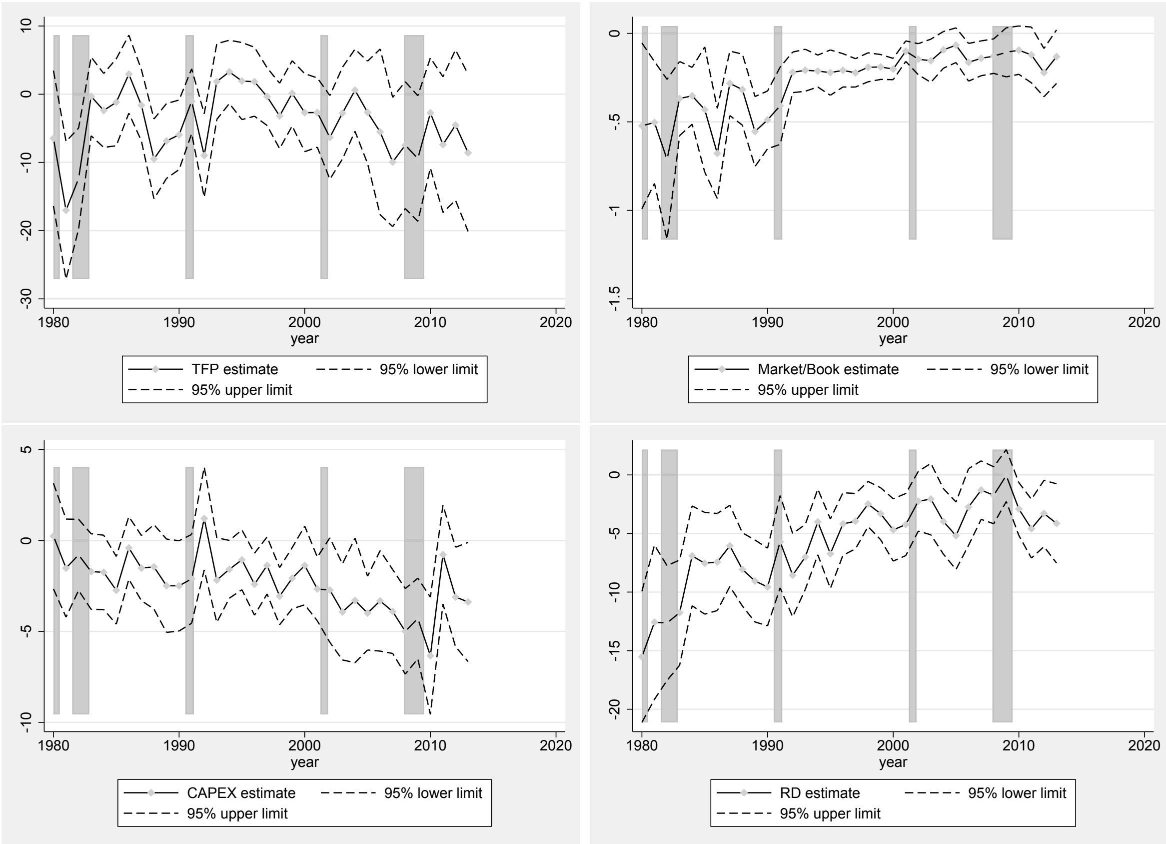

Figure 4.3: Repeated logit cross-section regressions estimates with 95% confidence boundaries corresponding to the effects of TFP, Market/Book and investment variables (CAPEX and R&D) on the firm's propensity to pay shareholders. The logit regressions are repeated for every year from 1980 to 2013 and they include all the growth indicators simultaneously. Controls common to all regressions include: market capitalisation, assets, cash, net income, debt and the number of employees. Grey areas indicate NBER recession periods.

their development are less likely to reward shareholders through cash distributions.[9] Moreover, the absence of dynamic effects of cash holdings makes it harder to interpret the results. In order to correct for these issues, I run a number of panel data fixed effects regressions explaining the size of the cash diverted towards shareholders.

4.3.3 Size of payout: evidence from dynamic fixed effects regressions

After considering the propensity of firms to divert any cash at all towards shareholders, I now turn to the size of the payout, expressed as a fraction of the previous period's market capitalisation. In order to capture the strong persistence of cash distributions and to control for non

[9]The negative correlation between the cash to asset ratio and the value of the firm's assets appears to give some credit to this explanation (table 4.8 of appendix 4.A.2).

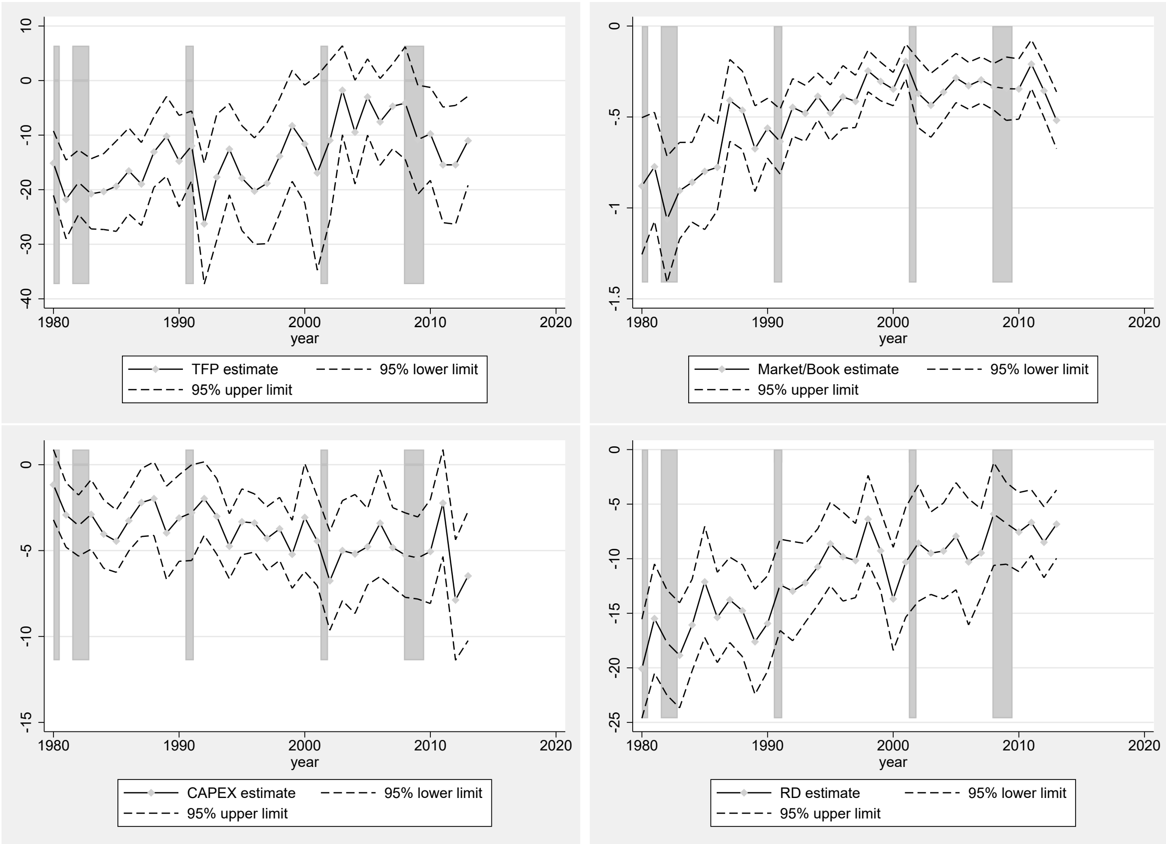

Figure 4.4: Repeated logit cross-section regressions estimates with 95% confidence boundaries corresponding to the effects of TFP, Market/Book and investment variables (CAPEX and R&D) on the firm's propensity to pay shareholders through **dividends**. The logit regressions are repeated for every year from 1980 to 2013. The TFP marginal effect is estimated without controlling for Market/Book, CAPEX and R&D, The Market/Book marginal effect is estimated without controlling for TFP, CAPEX and R&D and the CAPEX and R&D effects are estimated in the same repeated regressions that exclude both TFP and Market/Book. Controls common to all regressions include: market capitalisation, assets, cash, net income, debt and the number of employees. Grey areas indicate NBER recession periods.

time varying firm level characteristics, I exclude observations where no cash has been returned and run a two-way fixed effect dynamic panel data regression to explain the size of the cash returned to ordinary equity holders during the period between 1980 and 2013. Similarly to the static regressions case, each of the main growth indicators are included in a separate regression to assess its effect in absence of other indicators. I also present the results of a regression including all the growth indicators to show which ones maintain a significant effect in the presence of the others. The size of the overall cash distributed to shareholders is a strongly persistent process. This requires the inclusion of multiple lagged dependent variables in the regressions. To deal with the issue of estimating the coefficients of the lagged dependent variables, I exclude firms with less than 15 observations within the sample period, this guarantees that the average

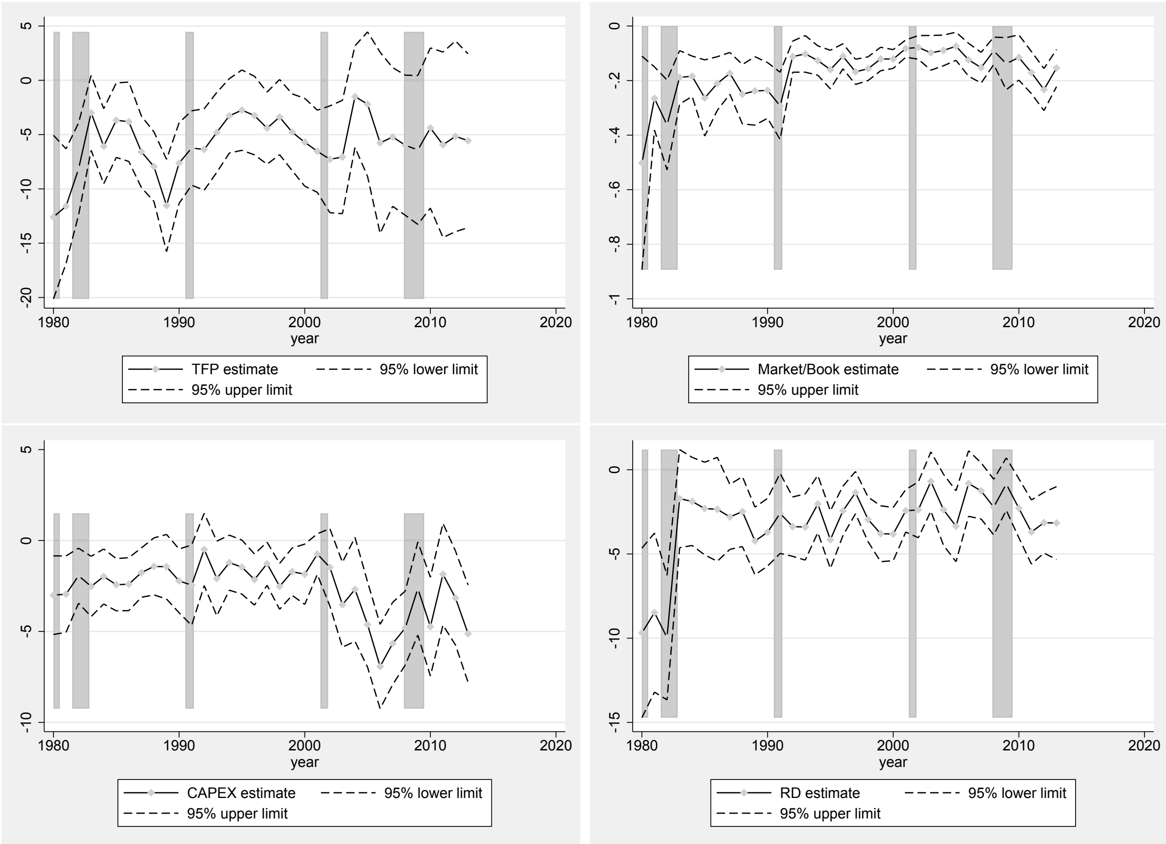

Figure 4.5: Repeated logit cross-section regressions estimates with 95% confidence boundaries corresponding to the effects of TFP, Market/Book and investment variables (CAPEX and R&D) on the firm's propensity to pay shareholders through **share buybacks**. The logit regressions are repeated for every year from 1980 to 2013. The TFP marginal effect is estimated without controlling for Market/Book, CAPEX and R&D, The Market/Book marginal effect is estimated without controlling for TFP, CAPEX and R&D and the CAPEX and R&D effects are estimated in the same repeated regressions that exclude both TFP and Market/Book. Controls common to all regressions include: market capitalisation, assets, cash, net income, debt and the number of employees. Grey areas indicate NBER recession periods.

number of time observations per firm is higher than 20 in all regressions.[10] The latter condition restricts the size of the sample substantially. In order to increase the sample size, I exclude the sparsely populated R&D variable from all regressions.[11] The model can be summarised as follows

$$y_t = \sum_{i=1}^{3} \beta_i y_{t-i} + \beta_x x_{t-1} + \beta_z z_{t-1} + \text{fixed effects} + \text{time dummies}, \tag{4.3.2}$$

[10]See Nickell (1981) and Bruno (2005) for more on the issue of estimating dynamic panel data regressions with a large number of units and a small number of observations per unit.

[11]Excluding R&D expenses for the repeated logit regressions does not affect the estimation results in a way that undermines the conclusion made in subsection 4.3.2.

where y_t is the distributed cash expressed as a fraction of the previous period's market capitalisation, x_{t-1} denotes the growth indicators of interest

$$x_{t-1} := \begin{cases} \text{TFP} \\ \boldsymbol{OR} \\ \text{Market/Book} \\ \boldsymbol{OR} \\ \text{CAPEX} \\ \boldsymbol{OR} \\ [\text{TFP, Market/Book, CAPEX}], \end{cases}$$

and z_{t-1} are the same controls as in the logit regressions presented in subsection 4.3.2.

The results in table 4.2 confirm the strong persistence of the size of the payout. Furthermore, all growth indicators have economically and statistically significant effects when other growth indicators are excluded. The Akaike information criteria show that the market to book ratio outperforms TFP and CAPEX as a growth indicator. This is confirmed by the results of the model that uses all growth indicators as explanatory variables (Full Model). In this model, the market to book ratio is the only growth indicator with a statistically significant coefficient at the 0.001 confidence level. The dynamic regressions' results confirm that large firms by assets' value tend to pay more relative to their market capitalisation with the estimated coefficient being statistically and economically significant and stable in value in all regressions. In the presence of the market to book ratio in the regression, the latter result is extended to large firms by market capitalisation. In the absence of the book to market ratio as an explanatory variable, net income has a negative and statistically significant effect on the distributed cash. This effect changes sign and becomes statistically insignificant in the presence of the market to book ratio. As predicted above, when controlling for the firm's idiosyncratic effects and taking dynamic aspects into account, firms with relatively high cash holding tend to pay shareholders more. This is consistent with the agency theory presented by Jensen (1986), stipulating that firms holding large sums of idle cash have an incentive to distribute more through dividends and share buybacks in order to reassure shareholders on the potential conflict of interest where corporate managers keep large cash amounts on the firm's balance sheet as a way to increase resources under their control. Finally, the size of the distributed cash decreases with the number of employees. This indicates that when controlling for the firm's fixed effects, changes in the number of employees represent a proxy for investment in the labour force.

	Full Model	TFP Effect	Market/Book Effect	Investment Effect
L.Dist. Cash	0.166***	0.181***	0.167***	0.180***
L2.Dist. Cash	0.0439***	0.0489***	0.0448***	0.0475***
L3.Dist. Cash	0.0423***	0.0461***	0.0433***	0.0449***
L.TFP	-0.0481	-0.122*		
L.Market/Book	-0.00447***		-0.00463***	
L.CAPEX	-0.0147**			-0.0239***
L.Market Cap. percentile	0.000176**	0.00000163	0.000187**	0.0000364
L.Assets	0.000000385***	0.000000493***	0.000000387***	0.000000453***
L.Cash	0.0310***	0.0268***	0.0314***	0.0256***
L.Net Income	0.0113	-0.0144*	0.00814	-0.0125*
L.Debt	-0.0182***	-0.0191***	-0.0192***	-0.0176***
L.Employees	-0.238***	-0.279***	-0.245***	-0.266***
Constant	0.0262***	0.0317***	0.0249***	0.0309***
AIC	-100793.7	-100567.1	-100786.5	-100585.9

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4.2: Two-way fixed effects dynamic model for the size of the **cash distributed**.

The dynamic fixed effect regression results for the size of dividends and share buybacks are in tables 4.3 and 4.4. The results show that both dividends and share buybacks sizes are persistent processes with dividends' size showing stronger persistence. When explaining the size of dividends and share buybacks separately from each other, TFP fails to have a statistically significant effect even when other growth indicators are excluded from the regression. This provides extra motivation to consider the combined "distributed cash" variable. Firms investing in capital expenditure tend to execute smaller share buybacks operation, while CAPEX investments do not appear to affect the size of dividends. Large firms by market capitalisation or assets tend to pay large dividends relative to their market capitalisation with assets' size having little impact on the size of dividends. On the other hand, large firms by assets are more likely to complete larger share buyback operations with little effect attributed to the market capitalisation percentile. Higher net income increases the size of dividends while not impacting the size of share buybacks operations. The results also suggest that firms use share buybacks more than dividends to manage relatively high cash balances. Finally, hiring reduces the size of both dividends and share buybacks relative to market capitalisation.

	Full Model	TFP Effect	Market/Book Effect	Investment Effect
L.Dividends	0.467***	0.476***	0.468***	0.477***
L2.Dividends	0.0888***	0.0889***	0.0882***	0.0892***
L3.Dividends	0.0460***	0.0464***	0.0456***	0.0469***
L.TFP	-0.0272	-0.0421		
L.Market/Book	-0.00129***		-0.00127***	
L.CAPEX	0.00334			0.000873
L.Market Cap. percentile	0.000115***	0.0000685**	0.000121***	0.0000769***
L.Assets	8.34e-08***	0.000000101***	7.69e-08**	9.52e-08***
L.Cash	0.00437**	0.00335*	0.00417**	0.00331*
L.Net Income	0.00501	-0.00102	0.00504	-0.00204
L.Debt	-0.00129	-0.00103	-0.00108	-0.00111
L.Employees	-0.0396*	-0.0482**	-0.0381*	-0.0490**
Constant	0.00388*	0.00562***	0.00365*	0.00501**
AIC	-144153.6	-144041.2	-144151.5	-144036.0

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4.3: Dynamic two way fixed effect model FE model explaining the size of **dividends**.

	Full Model	TFP Effect	Market/Book Effect	Investment Effect
L.Shares Buybacks	0.116***	0.129***	0.117***	0.127***
L2.Shares Buybacks	-0.00244	0.000751	-0.00152	-0.000444
L3.Shares Buybacks	0.0258	0.0291*	0.0269*	0.0275*
L.TFP	0.0657	-0.0198		
L.Market/Book	-0.00346***		-0.00358***	
L.CAPEX	-0.0226**			-0.0297***
L.Market Cap. percentile	0.000110	-0.0000460	0.0000927	-0.0000310
L.Assets	0.000000254**	0.000000376***	0.000000276**	0.000000352***
L.Cash	0.0301***	0.0257***	0.0310***	0.0251***
L.Net Income	0.00510	-0.0160*	0.00344	-0.0113
L.Debt	-0.0201***	-0.0214***	-0.0216***	-0.0193***
L.Employees	-0.272***	-0.317***	-0.288***	-0.298***
Constant	0.0273***	0.0327***	0.0275***	0.0331***
AIC	-49301.8	-49219.3	-49295.4	-49236.4

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4.4: Dynamic two way fixed effect model FE model explaining the size of **share buybacks**.

4.3.4 Summary of the empirical results

After considering empirical evidence linking the firm's appetite to grow to its propensity to pay shareholders and the size of the payouts, it appears that firms with an ability and appetite for growth divert less cash towards shareholders. I measure the appetite/ability to grow using the market to book ratio, investment expenses and TFP. While the market to book ratio performs better than other growth indicators in explaining distributions to shareholders, the TFP's marginal effect on the propensity to pay and size of the payouts is significant both in economic and statistic terms in the absence of other growth indicators.[12] As predicted by the theory, firms holding large sums of cash are more likely to divert cash towards shareholders. This relationship fails to appear in the repeated static logit regressions that do not control for the changes in cash levels and for the firm idiosyncratic effects, but is shown to hold in the two-way fixed effect dynamic regression explaining the size of shareholders payout.

The model developed in section 2.2 assumes that profit maximising firms can choose to distribute some of the cash at their disposal to shareholders instead of spending to produce, following unpredicted drops in productivity. The evidence presented above can serve as an empirical justification to model's mechanism linking the firm's productivity to the firm's spending.

4.4 Concluding remarks

I study the effect of productivity on the propensity of firms to pay shareholders back and on the size of these payouts. I show that higher firm level productivity lowers both the likelihood and the levels of the payouts. I assume that part of the cash diverted towards investors would have been spent on improving or increasing production had the firm decided against rewarding shareholders in the short term. The latter assumption and the negative empirical relationship between productivity and investor payouts indicate that firms decrease spending to respond to

[12]The market to book ratio is derived using the share price. It is reasonable to assume that market participants take into account information regarding the firm's productivity and investment expenses when setting their beliefs about the share price. Additionally, TFP is a noisy measure of the firm's production efficiency. It is therefore not surprising that market to book ratio is a superior measure of the firm's ability/wiliness to grow.

negative productivity shocks. The findings of this chapter serve as an empirical justification of the model presented in chapter 2.

4.A Appendix To Chapter 4

4.A.1 Data and derived variables

These variables are reported directly by the data: assets' value, cash and cash equivalent, net income, CAPEX spending, R&D spending and the number of employees. Other variables are derived as follows.

Market Capitalisation = "Common Shares Outstanding" × "Price Close - Annual - Calendar";

Debt = "Debt in Current Liabilities - Total" + "Long-Term Debt - Total";

Market to Book = "Market Capitalisation" / "Assets - Total";

Value Added = "Operating Income Before Depreciation" + "Employees" x "Average Wage from the Social Security Administration";

Share Buybacks = 1 year change in "Treasury Stock - Common", if the above is negative or missing use: "Purchase of Common and Preferred Stock" minus "Sale of Common and Preferred Stock". If both figures are negative or missing, Share Buybacks are set to zero for the corresponding period.

Except for the assets value, all the variables are scaled either using the percentile form of the variable or through division by the previous time period's assets or the previous market capitalisation. All the variables but those in percentile form are 1% Winsorised to deal with outlier values. These data transformations are summarised in table 4.5.

4.A.2 Descriptive Statistics

I present the summary statistics of the indicators used to construct the regressions variable in table 4.6, the summary statistics of the transformed variables are in table 4.7. The correlation matrix of the variables as used in the regressions are in table 4.8.

	Divided by prev. assets	Percentile	Divided by prev. market cap.	Winsorised (1%)
Assets				✓
Cash Dist.			✓	✓
Dividends			✓	✓
Share Buybacks			✓	✓
Productivity	✓			✓
Market to Book				✓
Market Cap.		✓		
Cash	✓			✓
Net Income	✓			✓
Debt	✓			✓
CAPEX	✓			✓
R&D	✓			✓
Employees	✓			✓

Table 4.5: Summary of transformation applied to the models' variables.

	mean	sd	min	p1	p25	p50	p75	p99	max
Cash Dist.	73.89	601.32	0.00	0.00	0.00	0.00	5.14	1503.00	67643.80
Dividends	47.02	389.75	0.00	0.00	0.00	0.00	2.04	982.00	67643.80
Shares Buy.	23.64	313.91	0.00	0.00	0.00	0.00	0.00	461.59	34420.00
TFP	1.21	2.23	0.00	0.13	0.82	1.03	1.28	4.21	283.51
Market Cap.	2256.37	13201.11	0.00	0.67	21.74	107.18	619.62	43294.66	1819781.88
CAPEX	148.67	977.18	-401.61	0.00	0.77	5.20	34.44	2760.00	65028.00
RD	85.50	487.32	-0.55	0.00	0.19	2.72	17.11	2015.00	14035.29
Assets	2200.24	13328.37	0.50	0.98	20.94	106.47	601.96	39042.00	797769.00
Cash	212.67	1406.27	-40.00	0.00	1.25	8.54	52.37	4007.00	91052.00
Net Income	91.63	1056.33	-98696.00	-353.71	-2.32	1.39	18.78	2285.29	125000.00
Debt	668.75	5355.19	0.00	0.00	1.67	16.85	168.75	11122.70	523762.00
Employees	9.21	37.64	0.00	0.00	0.16	0.85	4.35	144.78	2200.00

Table 4.6: Summary statistics of the unscaled data used to construct the dependent and independent variables used in the various regressions: all cash variables are in millions of U.S. dollars, the number of employees is in thousands, data for the 1980-2013 period.

4.A.3 More Empirical Results

Further empirical results are presented in this subsection. I present the complete results of the logit regressions including all growth indicators results and explaining, respectively, the propensity to return cash, to pay dividends and to buy shares back in figures 4.6 to 4.8.

	mean	sd	min	p1	p25	p50	p75	p99	max
Cash Dist.	0.02	0.04	0.00	0.00	0.00	0.00	0.02	0.24	0.24
Dividend	0.01	0.02	0.00	0.00	0.00	0.00	0.01	0.12	0.12
Share Buy.	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.17	0.17
TFP	0.02	0.04	0.00	0.00	0.00	0.00	0.01	0.23	0.23
Market/Book	1.78	2.75	0.05	0.05	0.45	0.90	1.84	17.17	17.17
CAPEX	0.09	0.13	0.00	0.00	0.02	0.05	0.10	0.76	0.76
RD	0.08	0.13	0.00	0.00	0.00	0.03	0.11	0.72	0.72
Mkt. Cap. %	39.70	22.70	1.00	1.00	20.00	40.00	59.00	80.00	85.00
Assets	1513.10	4736.80	0.67	0.98	20.94	106.47	601.96	31001.40	31001.40
Cash	0.19	0.31	0.00	0.00	0.02	0.08	0.22	1.81	1.81
Net Income	-0.04	0.28	-1.50	-1.50	-0.06	0.03	0.08	0.43	0.43
Debt	0.29	0.26	0.00	0.00	0.08	0.24	0.41	1.37	1.37
Employees	0.01	0.02	0.00	0.00	0.00	0.01	0.02	0.10	0.19

Table 4.7: Summary statistics of the scaled variables used in the regression models, data for the 1980-2013 period.

	Cash Dist.	Div.	Shares Buy.	TFP	Mkt/Book	CAPEX	R&D	Mkt Cap.	Assets	Cash	Net Inc.	Debt	Empl.
Cash Dist.	1.00	0.65	0.75	-0.13	-0.15	-0.07	-0.18	0.15	0.13	-0.12	0.13	-0.00	-0.03
Div.	0.65	1.00	0.05	-0.16	-0.14	-0.05	-0.21	0.24	0.19	-0.15	0.15	0.01	-0.02
Shares Buy.	0.75	0.05	1.00	-0.06	-0.10	-0.07	-0.10	0.05	0.05	-0.06	0.07	-0.02	-0.02
TFP	-0.13	-0.16	-0.06	1.00	0.24	0.09	0.17	-0.47	-0.17	0.20	0.07	-0.01	0.24
Mkt/Book	-0.15	-0.14	-0.10	0.24	1.00	0.25	0.50	0.15	-0.07	0.63	-0.21	-0.06	0.10
CAPEX	-0.07	-0.05	-0.07	0.09	0.25	1.00	0.09	0.08	-0.04	0.13	-0.01	0.26	0.17
R&D	-0.18	-0.21	-0.10	0.17	0.50	0.09	1.00	-0.08	-0.12	0.58	-0.50	-0.16	-0.08
Mkt. Cap.	0.15	0.24	0.05	-0.47	0.15	0.08	-0.08	1.00	0.46	0.02	0.27	0.02	-0.12
Assets	0.13	0.19	0.05	-0.17	-0.07	-0.04	-0.12	0.46	1.00	-0.09	0.10	0.03	-0.15
Cash	-0.12	-0.15	-0.06	0.20	0.63	0.13	0.58	0.02	-0.09	1.00	-0.24	-0.14	0.04
Net Inc.	0.13	0.15	0.07	0.07	-0.21	-0.01	-0.50	0.27	0.10	-0.24	1.00	-0.02	0.06
Debt	-0.00	0.01	-0.02	-0.01	-0.06	0.26	-0.16	0.02	0.03	-0.14	-0.02	1.00	0.09
Empl.	-0.03	-0.02	-0.02	0.24	0.10	0.17	-0.08	-0.12	-0.15	0.04	0.06	0.09	1.00

Table 4.8: Correlation matrix of regressions' variables (1980-2013).

4.A.4 Tests

The studied dependent variables follow strongly persistent processes. Failing to correct for such persistence can cause serial correlation tests to fail. I present the serial correlation test in table 4.10. The tests show that serial correlation is either statistically insignificant or too low to seriously affect the result of the regressions.

Running dynamic panel data models for a large number of units and a small number of observations per unit comes with the issue of a biased estimate of the coefficient of the lagged dependent variables. The absence of an important serial correlation in the error terms provides an indication that there is little underestimation of the lagged variables coefficient if any. To gain more confidence around this issue, regressions are run where the number of observations per firm is unrestricted, is required to be higher than 30 ($T \geq 30$) (table 4.11) . The results show that, as expected by the theory,

	Full Model	TFP Effect	Market/Book Effect	Investment Effect
L.Dist. Cash	0.0830***	0.0939***	0.0839***	0.0938***
L2.Dist. Cash	0.00819	0.0120	0.00882	0.0105
L3.Dist. Cash	0.00273	0.00468	0.00363	0.00407
L.TFP	-0.0824**	-0.158***		
L.Market/Book	-0.00443***		-0.00470***	
L.CAPEX	-0.0138***			-0.0244***
L.MArket Cap. percentile	0.000208***	-0.0000430	0.000230***	0.0000208
L.Assets	0.000000425***	0.000000573***	0.000000413***	0.000000510***
L.Cash	0.0323***	0.0237***	0.0324***	0.0221***
L.Net Income	0.0199***	0.00491	0.0161***	0.00305
L.Debt	-0.0170***	-0.0184***	-0.0182***	-0.0163***
L.Employees	-0.218***	-0.268***	-0.234***	-0.269***
Constant	0.0328***	0.0418***	0.0310***	0.0394***
AIC	-201928.0	-201460.3	-201897.9	-201460.1

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4.9: Dynamic two way fixed effect model FE model explaining the size of distributed cash, no exclusion of firms based on the number of observations.

a low number of observation per unit leads to underestimating the autoregressive coefficients. The differences in the lagged dependent variables estimates remain small when increasing the minimum number of observations per firm from 15 to 30.

	Cash Dist. Resid.	Div. Resid.	Shares Buy. Resid.
L.residuals	-0.0234*	-0.0189	0.0173
L2.residuals	-0.0176	-0.00106	0.0215*
L3.residuals	-0.0184	-0.00385	-0.00282
L4.residuals	-0.0150	-0.00580	-0.0132
L5.residuals	-0.0263**	-0.0153	-0.0220**
L6.residuals	-0.0201*	-0.000362	-0.00750
L7.residuals	-0.0178*	0.00615	-0.0253**
Constant	0.000251***	-0.000524***	-0.0128***
Observations	20660	20660	20738

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4.10: Autocorrelation tests (Full Models).

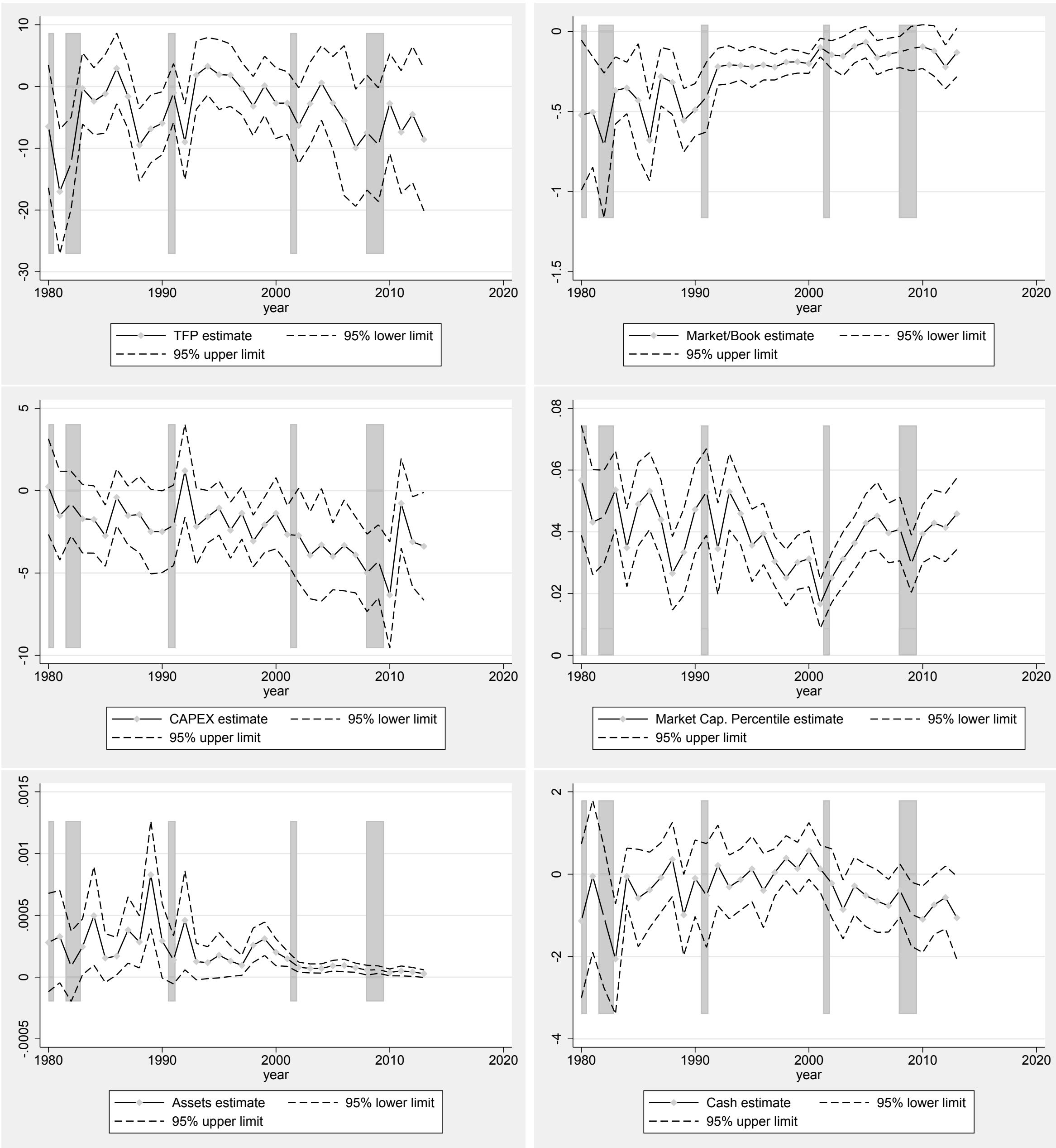

Figure 4.6: Repeated logit cross-section regressions estimates with 95% confidence boundaries for the full model explaining the **distributed cash** (continued below).

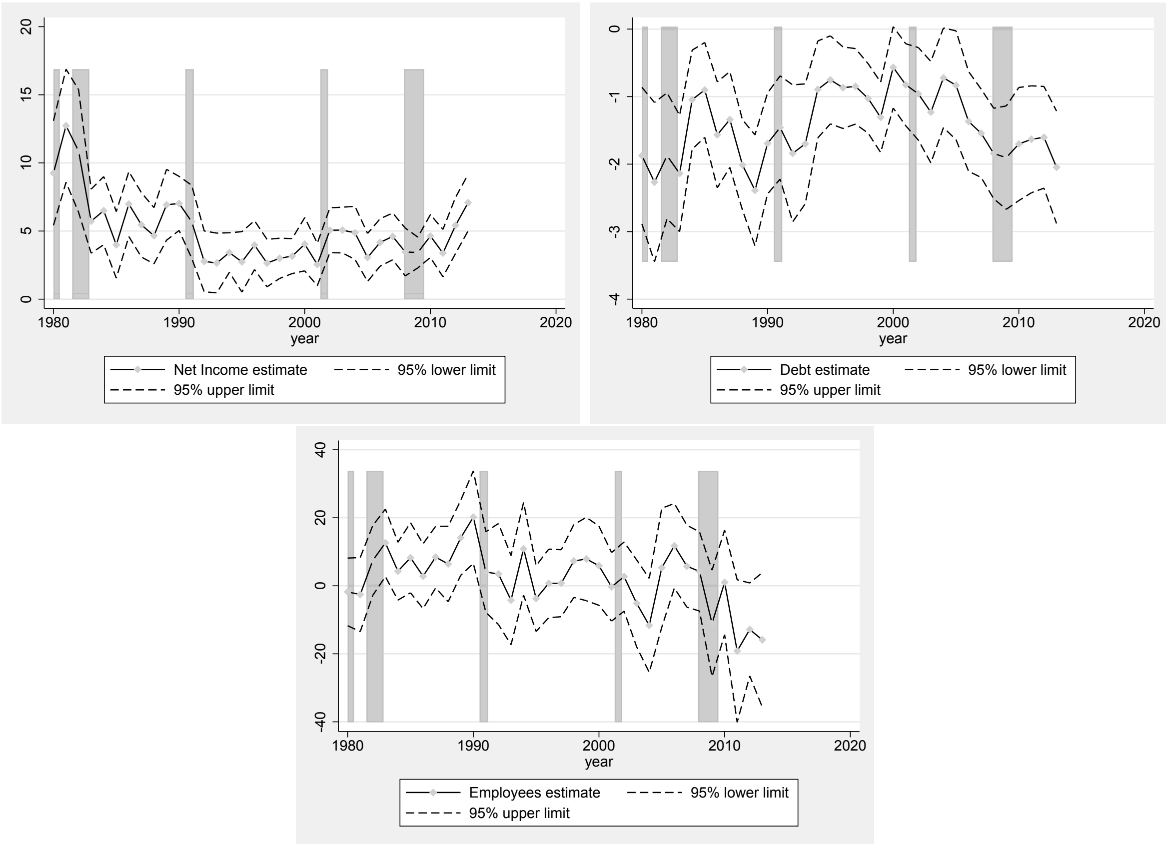

Figure 4.6: Repeated logit cross-section regressions estimates with 95% confidence boundaries for the full model explaining the **distributed cash**.

	Cash Dist	T $\geq$ 15	T $\geq$ 30	Div	T$\geq$ 15	T $\geq$ 30	Share Buy	$\geq$ 15	T $\geq$ 30
L.Dist. Cash	0.083***	0.17***	0.19***						
	(0.0079)	(0.010)	(0.017)						
L2.Dist. Cash	0.0082	0.044***	0.052***						
	(0.0068)	(0.0080)	(0.013)						
L3.Dist. Cash	0.0027	0.042***	0.059***						
	(0.0071)	(0.0083)	(0.015)						
L.Dividends				0.37***	0.47***	0.47***			
				(0.013)	(0.017)	(0.033)			
L2.Dividends				0.066***	0.089***	0.13***			
				(0.011)	(0.014)	(0.027)			
L3.Dividends				0.027**	0.046***	0.062**			
				(0.0085)	(0.010)	(0.021)			
L.Share Buybacks							0.028**	0.12***	0.10***
							(0.010)	(0.014)	(0.023)
L2.Share Buybacks							-0.032***	-0.0024	0.016
							(0.0098)	(0.013)	(0.020)
L3.Share Buybacks							-0.023*	0.026	0.018
							(0.011)	(0.014)	(0.019)
Observations	53104	25298	8971	39786	23474	8902	29393	12282	4474

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4.11: Lagged dependent variables tests. The lagged variables coefficients from the model including all firms are shown next to estimates of the same coefficients from a models excluding firms with less than 15 observations and 30 observations respectively.

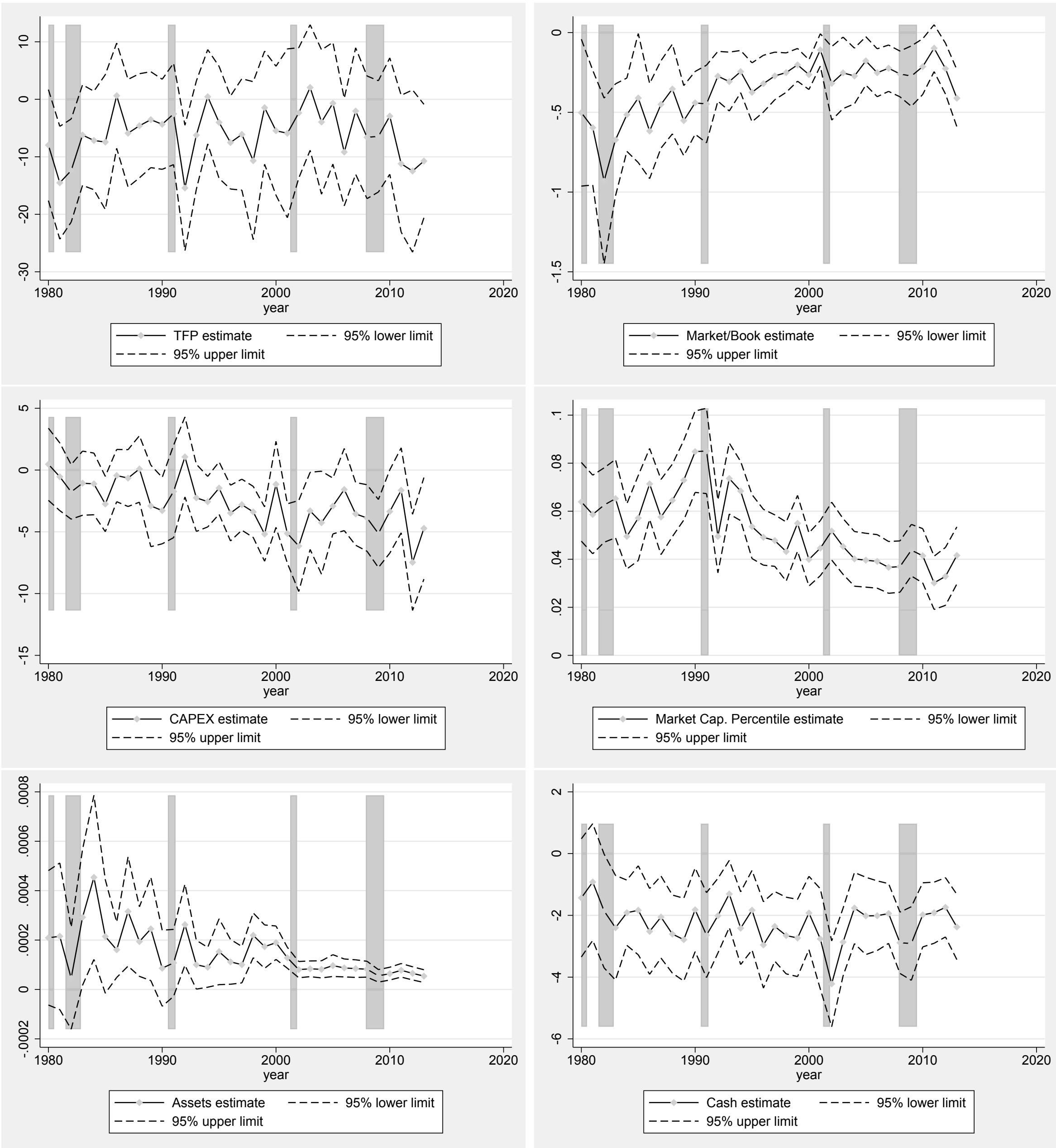

Figure 4.7: Repeated logit cross-section regressions estimates with 95% confidence boundaries for the full model explaining the propensity to pay **dividends** (continued below).

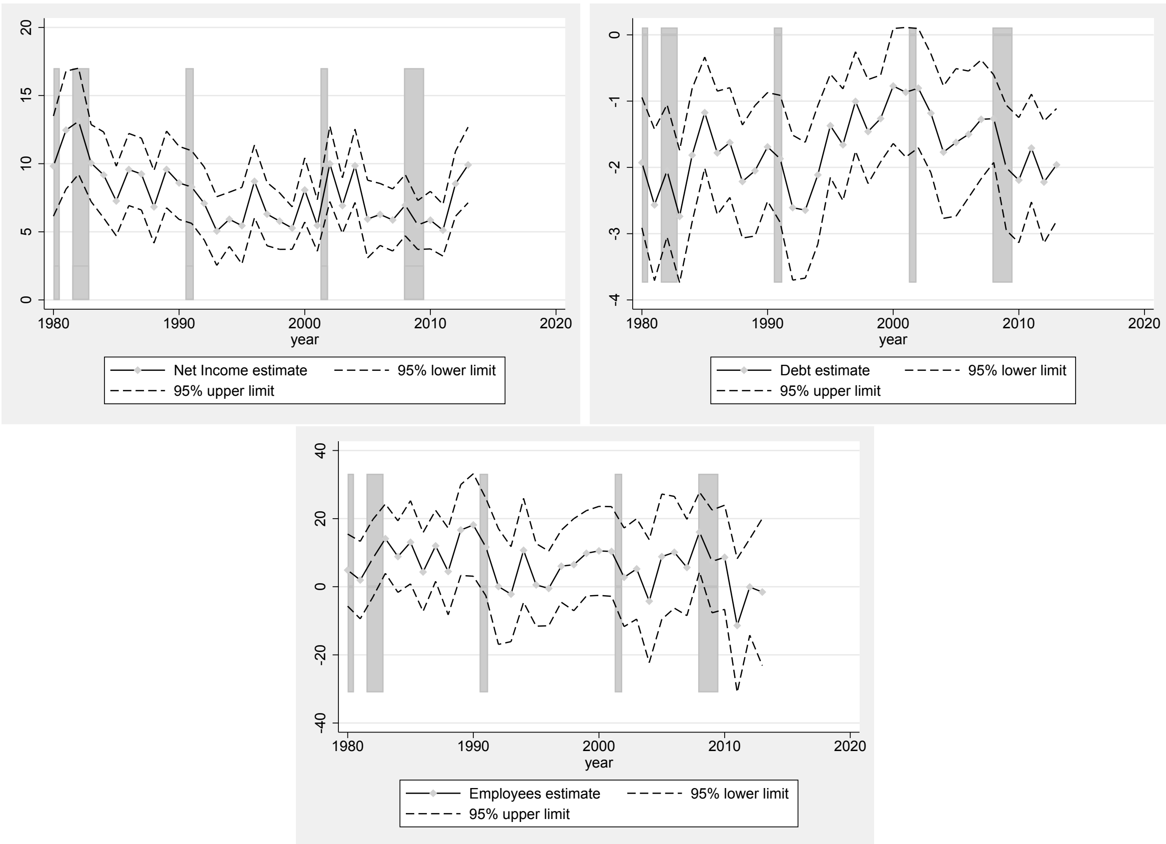

Figure 4.7: Repeated logit cross-section regressions estimates with 95% confidence boundaries for the full model explaining the propensity to pay **dividends**.

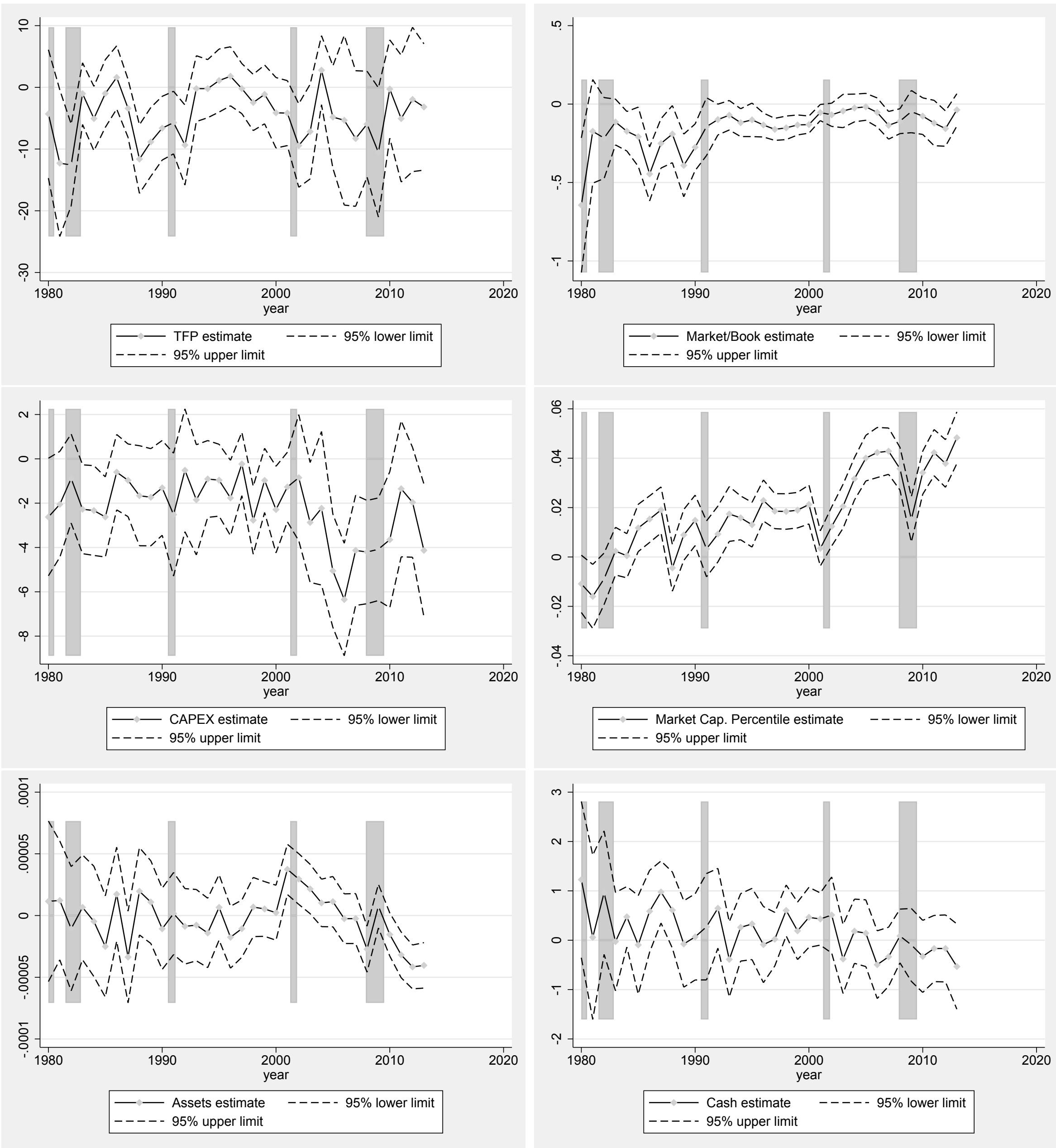

Figure 4.8: Repeated logit cross-section regressions estimates with 95% confidence boundaries for the full model explaining **share buybacks** (continued below).

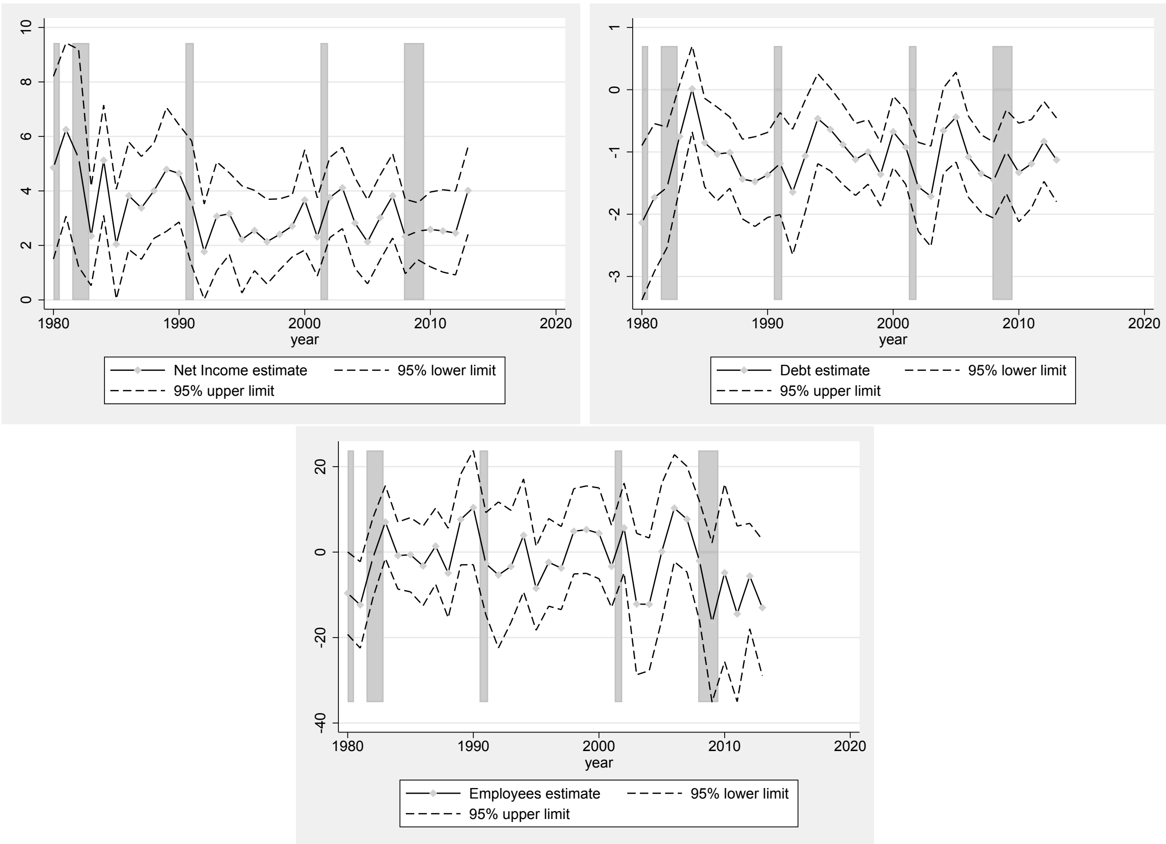

Figure 4.8: Repeated logit cross-section regressions estimates with 95% confidence boundaries for the full model explaining **share buybacks**.

Chapter 5

Credit Markets, Intermediate Production and the Business Cycle

5.1 Introduction

I present a dynamic stochastic general equilibrium model with a sector of firms that borrow to finance the production of a single intermediate good, banks providing the required financing and an endogenous mechanism for the borrowing firms to bankrupt on their debt obligations. The intermediate good is used as an input by a representative firm that hires labour and rents capital to produce a final good that is then sold to households. Within the studied framework, intermediate production firms raise financing using the information available to them at the financing stage, including how they assess their own production efficiency and the expected sale prices of the intermediate good they produce. When a borrowing firm realises its actual production efficiency and the prevailing sale prices, its revenues might drop to levels below initial expectations. If this drop is large enough, the firm's revenue may not suffice to repay the debt obligations the firm committed to at the financing stage. The firm would then fail to honour its financial obligation. As a result, the lending bank takes over the firm, and a fraction of production is lost to reflect the costs of the bankruptcy workouts and the fact that banks are less knowledgeable about the production process.

Intermediate production firms sell the good they produce to a representative final producer that is constrained by a Cobb-Douglas type technology and using labour, capital and the intermediate good as inputs. The productivity of the intermediate production firms is decomposed into an idiosyncratic part that is specific to the firm and a systemic component that is shared with the productivity of the final good producing firm. This assumption guarantees two desirable outcomes of the model. First, bankruptcy is limited to a subset of the intermediate production firms, namely the group of firms with low enough productivities. Second, the importance of the systemic part of the intermediate producers' efficiencies helps control the intermediate production fluctuations' contribution to the variance of key aggregate variables.

The division of the productive sector into one group of firms that borrows to produce an intermediate good and a final representative firm that uses the intermediate good as input is crucial for the model's main mechanism to operate. The intermediate producers' output is an input in the final production process; its price displays a procyclical behaviour. When aggregate productivity, defined as the final producer's productivity, is high, demand for the intermediate good increases, and so does its price. Higher sale prices improve the revenues of the borrowing firms and, in turn, decreases default rates. Similarly, revenues are depressed when aggregate productivity is deteriorating so that default rates are higher and credit spreads are wider. This way, countercyclical default rates and credit spreads are generated even when one assumes no correlation between intermediate and aggregate production efficiencies.

While not crucial in generating countercyclical default rates and credit spreads, other model assumptions are useful in generating realistic second moments of credit spreads and important aggregate variables. For instance, I assume that intermediate production firms are subject to a quadratic cost of changing the production size. This limits the changes in loans' demand by the intermediate production firms following fluctuations in total factor productivity. The borrowing firm's inability to change its production levels without incurring a cost maintains borrowing at relatively high levels during economic slumps. As a result, default rates are even higher in periods of recession. Higher default rates cause wider credit spreads during periods of recession as banks need to hike loan rates to reflect higher default expectations. Similarly, this friction causes lower default rates, and tighter credit spreads when aggregate productivity is rising. While the intermediate/final production structure is key to generating countercyclical defaults rates and credit spreads, the costly change of the borrowing firm production level helps reproduce quantitatively realistic dynamics of credit spreads.

Banks are assumed to be competitive and face no entry costs. Therefore, in expectation, banks make no profit and no loss from extending loans. To compensate for the future default of a fraction of the intermediate production firms, banks charge a credit spread on the top of the interest rate they expect to pay depositors. When macroeconomic conditions are worse, both the intermediate good's price and the intermediate production efficiency are lower. This depresses the revenues of the borrowing firms and, as a result, causes the proportion of firms expected to default to increases. Banks take this into account and raise the loans' interest rates. The business cycle is depressed further, in a way reminiscent of the financial accelerator effect studied by Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1998) and other authors. The framework studied in this paper focuses on the way banks readjust the interest rate they charge firms to reflect the relationship between business cycles and bankruptcy rates, while typical financial accelerator models assume rationing of credit following a deterioration in the market value of assets used as collateral by borrowers.

Non-exhaustive literature review– There is a rich literature on general equilibrium models with endogenous bankruptcies. Carlstrom and Fuerst (1997) build on the Bernanke and Gertler (1989) model whereby lending agency costs arise endogenously and introduce a class of capital transforming entrepreneurs who rely on their net-worth as collateral to raise external debt financing. These entrepreneurs can default on their debt obligations after a large enough negative shock affects their ability to transform capital. The possibility of defaults interacts with the dynamics of the entrepreneurs' net-worth over the business cycle to replicate the empirically observed positive autocorrelation of output at short horizons. In this paper, I ignore the net-worth effects on the ability of borrowers to secure external financing. Instead, I link default probabilities to the aggregate TFP driving the business cycle–Mostly through the variation of the intermediate good prices. This generates countercyclical financing costs that increase output when aggregate TFP improves and reduces it when TFP is lower. On the other hand, the costly financing general equilibrium model in Carlstrom and Fuerst (1997) fails to reproduce the empirically observed countercyclical default premiums (As noted in Gomes, Yaron, and Zhang (2003)).

The bankruptcy mechanism used in this paper is similar to the one in Pesaran and Xu (2016). A first difference resides in the way both models deal with employment. While Pesaran and Xu (2016) choose a specific consumer utility function to disentangle the problem of default from that of labour, I choose to dissociate defaults from labour demand by assuming that some firms borrow to produce an intermediate good (intermediate producers) and others hire labour, rent capital and use the intermediate good to produce the final good (final producers). Another difference lies in the banking sector's modelling, a mere multiplier in Pesaran and Xu (2016); the model presented here assumes that banks set interest rates taking expected profits into account. Banks provide the intermediate good producing firms with loans carrying an interest rate that reflects default expectations. Finally, intermediate productivity is assumed to be correlated with an aggregate total productivity factor driving final production. Unlike in Pesaran and Xu (2016) where default probabilities do not fluctuate with the cycle, the fact that the revenues of intermediate producers fluctuate with the cycle implies that default rates fluctuate with the cycle in a countercyclical fashion. Banks reflect expectations over future defaults in the interest rates they charge borrowers. This bank behaviour means that credit spreads are countercyclical. The countercyclical behaviour of credit spreads accelerates growth during the highs of the business cycle and through recession periods.

Christiano, Motto, and Rostagno (2010) develop a New Keynesian DSGE model with a mechanism for endogenous defaults. This is achieved by assuming that a class of entrepreneurs transforms capital by investing their net worth as a source of self-financing and securing the rest of the required financing through bank loans. Similarly to what I do in this paper, the authors assume that the efficiency of the entrepreneur's ability to transform capital is subject to idiosyncratic shocks. A sufficiently unfavourable shock can lead to the borrower's bankruptcy. Furthermore, the authors assume that the variance of the idiosyncratic shocks that affect the entrepreneur's return is the realisation of a time-varying process. As this variance changes through time, the cross-sectional distribution of returns also changes, thus producing time variation in credit risk and thereby time-varying credit spreads. In the model I consider here, the variations in default rates and credit spreads are generated by the borrowers' revenues' procyclical behaviour. This guarantees a countercyclical behaviour of bankruptcy rates and credit spreads without having to assume the existence of an additional exogenous process driving credit risk.

A different strain of literature attributes countercyclical credit premiums to sticky lender/borrower relationships. Aliaga-Diaz and Olivero (2010) and Aksoy, Basso, and Coto-Martinez (2013) argue that banks exploit existing lending relationships and the preference of borrowers to stay with the same lender and charge higher credit spreads during slowdowns. The authors build on the deep habits framework in Ravn, Schmitt-Groh, and Uribe (2006) to model the costs of bank switching.

Falato and Xiao (2020) argue that learning from noisy information is an important propagation mechanism for understanding credit and business cycles. They build a general equilibrium model with information asymmetries between lenders and borrowers whereby lenders interpret deteriorations in short-term profit outlook as bad news about default risk. In turn, firms perceive debt as underpriced and cut investments. The authors develop a model of credit-market investors' learning that generates countercyclical default rates and credit spreads. Their model can also quantitatively account for the long-lasting widening in spreads and contraction in aggregate investment during the 2007-09 financial crisis. The model I develop in this paper requires no asymmetries in the information available to the lender and borrower at the time of issuance. Countercyclical default rates and credit spreads are generated by the procyclical behaviour of the intermediate good's price. Procyclical prices guarantee that revenues are procyclical and are sufficient to make default rates move inversely with the cycle. Because credit spreads reflect the cost of future defaults, they too display countercyclical behaviour.

The remainder of this work is organised as follows. Section 5.2 presents the general equilibrium model. Section 5.3 provides details about the calibration and simulation of the model and comments the steady-state results and dynamic effects of the main mechanism. Section 5.4 concludes.

5.2 General equilibrium

I study a general equilibrium model with an intermediate good producing sector that provides a final representative firm with input. Intermediate production is funded through debt financing provided by a competitive banking sector with no entry costs that use households' deposit to fund its lending operations. Some of the borrowing firms can default on their debt obligations when their revenues are too low to cover debt payments. To achieve no profit and no loss in expectation, the competitive banks reflect the default losses they expect in the interest rates they charge the borrowing firms. The representative final producer uses capital and labour to produce the final good. This final good is either consumed by households, transformed into new capital or used as an input to the intermediate production process. Households set their consumption, labour supply, capital investments and deposits to maximise expected utility, discounted over their lifetime.

5.2.1 Intermediate good producing firms

I assume the existence of a sector with firms that transform output before its use as an intermediate good by final production firms. The intermediate production process is fully financed by banks through loan issuance. It takes an intermediate good producing firm a single time period to produce a quantity y^M_{t+1}, to do so it issues a single time period maturity loan to the banks of principal X_t and uses the production function below to transform the invested quantity of final good X_t into intermediate good y^M_{t+1}

$$y^M_{i,t+1} = Z_{i,t+1} f(X_{i,t}) \left(1 - \lambda(\frac{X_{i,t}}{X_{i,t-1}} - 1)^2\right), \tag{5.2.1}$$

where the index i denotes the firm, $Z_{i,t+1}$ is a heterogeneous and stochastic efficiency factor, λ is a parameter reflecting the cost of changing the level of production and $f(.)$ represents the deterministic component of the intermediate production technology before incurring adjustment costs. Furthermore, I assume that intermediate production efficiencies follow a log-normal processes

$$z_{i,t} := \ln Z_{i,t} = \sigma_M[\rho_M z_t^a + \sqrt{1-(\rho_M)^2}\epsilon_i]. \tag{5.2.2}$$

z_t^a is a common efficiency factor, representing aggregate productivity, that is time-dependent and that follows an AR(1) process: $z_t^a = \rho_a z_{t-1}^a + e_t$ where e_t are normal and *i.i.d.* shocks. The terms ϵ_i reflect the idiosyncratic firm efficiencies and are normally distributed, independent across firms and independent from the common efficiency factor z_t^a. The parameter σ_M is a volatility parameter representing the intermediate production projects' riskiness, and ρ_M reflects the interdependence of intermediate production across different firms. The type of firm ϵ_i is unknown before the loan's maturity so that all intermediate production firms are a priori identical, and they all face the same cost of financing R_t and raise the same loan principal X_t. They also produce the same intermediate good, sold at the common price Q_t.

In the case where the intermediate production firms do not default, their profit is

$$\pi_{i,t+1}^M = Q_{t+1}Z_{i,t+1}g(X_t, X_{t-1}) - R_t X_t,$$

where by definition $g(X_t, X_{t-1}) := f(X_t)\left(1 - \lambda(\frac{X_t}{X_{t-1}} - 1)^2\right)$. The intermediate good producing firms walk away on their debt if their non-default profit is negative. In the case where a firm defaults, the lending bank takes over intermediate good production and loses a fraction θ of the produced intermediate goods in the process, reflecting a cost for the bank to go through bankruptcy workouts and the fact that the firms' managers possess more knowledge about the production process than banks. In the spirit of Carlstrom and Fuerst (1997), an intermediate production firm chooses to default when its profit becomes negative, i.e. when $Z_{i,t+1} < \frac{R_t X_t}{Q_{t+1}g(X_t, X_{t-1})}$. In other words, defaults happen when standardised version of the Gaussian variable $z_{i,t+1}$ is below a certain value, denoted $-\xi_{t+1}$.[1] The default probability of the

[1]The standardised version of Gaussian variable $X \sim N(\mu, \sigma^2)$ is $\frac{X-\mu}{\sigma}$.

intermediate production firms is given by

$$DP_{t+1} = \Phi(-\xi_{t+1}), \tag{5.2.3}$$

where Φ denotes the normal cumulative distribution function. Following the nomenclature inspired by Merton (1974), ξ_{t+1} is called the "distance to default". In the current set-up, the distance to default depends on the common factor z^a_{t+1}, the current loan and previous loan sizes (X_{t-1}, X_t), the gross rate of interest R_t and the sale price Q_{t+1}

$$\xi_{t+1} = \frac{1}{\sqrt{1-\rho_M^2}} \left\{ \rho_M z^a_{t+1} + \frac{1}{\sigma_M} \ln\left(\frac{Q_{t+1} g(X_t, X_{t-1})}{R_t X_t} \right) \right\}. \tag{5.2.4}$$

Note here that DP_{t+1} is the default probability at the loan's maturity before the firm realizes its type ϵ_i. Once the type ϵ_i is known to the borrowing firm, default or survival is immediately determined and not random anymore. The used correlation structure and the assumption that $0 < \rho_M < 1$ guarantee a positive correlation of efficiencies across firms. During recessions, the common factor z^a_{t+1} is low, and all firms have lower efficiencies, while intermediate producers' efficiency is high when z^a_{t+1} is high. Aggregate TFP has a direct impact on default rates through the intermediate productivities channel. In addition to the direct effect of aggregate efficiency, indirect effects operate through other variables affecting the "distance to default". These variables are the price of the intermediate good Q_{t+1}, the size of the loan X_t and the charged interest rate R_t. While it is clear that, everything else being equal, default rates increase with higher loan levels X_t and higher interest rates R_t and decrease with higher sale prices Q_{t+1}, the net combined effect of these variables is unclear at this stage. Section 5.3 is dedicated to numerical simulations; it clarifies the magnitude of each of the competing effects and shows the net effect of aggregate fluctuations on default rates. Section 5.3 also shows the effect of the cost of adjusting investment as reflected by the parameter λ. The fact that changing the level of intermediate production is costly has two crucial implications. First, it dampens the changes in the levels of intermediate production. Second, the costly adjustment in intermediate production levels pushes intermediate producers to maintain a relatively high demand for loans when the productivity is low. The loan market clears at a higher interest rate R_t, implying higher credit spreads during recession periods. This and the fact that investment adjustment costs do not impact the steady-state of the model implies that the parameter λ is key to generating realistic credit spreads dynamics.

The intermediate goods' producing firms maximise profit, taking defaults into account, to set their demand of loans X_t

$$\max_{X_t} \mathbf{E}_t \left[\pi^{M,d}_{i,t+1} \right], \tag{5.2.5}$$

where $\mathbf{E}_t$ denotes the expectation operator conditional on the information set available at time t and $\pi^{M,d}_{i,t+1}$ is the profit taking into account the possibility of future default

$$\pi^{M,d}_{i,t+1} = Q_{t+1} Z_{i,t+1} g(X_t, X_{t-1}) \mathbf{1}_{\epsilon_i > -\xi_{t+1}} - R_t X_t \mathbf{1}_{\epsilon_i > -\xi_{t+1}}. \tag{5.2.6}$$

Given the independence between the idiosyncratic shocks ϵ_i and the aggregate productivity common factor z^a_{t+1}, one can rewrite expected profits as follows

$$\mathbf{E}_t \pi^{M,d}_{i,t+1} = g(X_t, X_{t-1}) e^{\Sigma_M^2/2} \mathbf{E}_t \left[Q_{t+1} e^{\rho_M \sigma_M z^a_{t+1}} \Phi(\xi_{t+1} + \Sigma_M) \right] - R_t X_t \mathbf{E}_t \Phi(\xi_{t+1}), \tag{5.2.7}$$

where $\Sigma_M := \sigma_M \sqrt{1 - \rho_M^2}$ is the volatility of the idiosyncratic part of the intermediate production efficiency. One can therefore write the loan demand equation in a form that is common to all firms and where the idiosyncratic shock ϵ_i plays no role

$$\begin{aligned} R_t \mathbf{E}_t \left[\Phi(\xi_{t+1}) + X_t \frac{\partial \xi_{t+1}}{\partial X_t} \phi(\xi_{t+1}) \right] &= \frac{\partial g\left(X_t, X_{t-1}\right)}{\partial X_t} e^{\Sigma_M^2/2} \mathbf{E}_t \left[Q_{t+1} e^{\rho_M \sigma_M z^a_{t+1}} \Phi(\xi_{t+1} + \Sigma_M) \right] \\ &+ g\left(X_t, X_{t-1}\right) e^{\Sigma_M^2/2} \mathbf{E}_t \left[Q_{t+1} e^{\rho_M \sigma_M z^a_{t+1}} \frac{\partial \xi_{t+1}}{\partial X_t} \phi(\xi_{t+1} + \Sigma_M) \right]. \end{aligned} \tag{5.2.8}$$

The latter formulation of the loan demand equation confirms the fact that all firms face the same gross interest rate R_t and raise the same level of loan X_t. Writing $\frac{\partial g(X_t, X_{t-1})}{\partial X_t}$ in the form below clarifies the role of the parameter λ in moderating the fluctuations of the loan demand

$$\frac{\partial g\left(X_t, X_{t-1}\right)}{\partial X_t} = f'(X_t)\frac{g\left(X_t, X_{t-1}\right)}{f\left(X_t\right)} - 2\frac{\lambda}{X_{t-1}^2}\left(X_t - X_{t-1}\right) f(X_t). \tag{5.2.9}$$

The parameter λ shifts loan demand lower when it is increasing in comparison to the previous period loan principal ($X_t > X_{t-1}$) and shifts it higher when it is decreasing ($X_t < X_{t-1}$).

5.2.2 Production firms

Production of the final good is performed by a representative firm that is constrained by a Cobb-Douglass technology that uses labour (H_t), capital (K_t) and the intermediate good (M_t) as inputs

$$Y_t = \left(Z_t^a H_t^{1-\alpha} K_t^{\alpha}\right)^{1-\zeta} M_t^{\zeta}, \tag{5.2.10}$$

where ζ is the share of the intermediate good and $(1-\zeta)\alpha$ is the share of capital. The production sector productivity Z_t^a is completely driven by the systemic factor defined in the previous section $\ln(Z_t^a) = \sigma_a z_t^a$. The demand for capital, labour and the intermediate input are set to maximize the final producer's profit

$$\max_{H_t, K_t, M_t} Y_t - r_t^K K_t - w_t H_t - Q_t M_t, \tag{5.2.11}$$

where w_t is the wage and r_t^K is the rental cost of labour. The first order condition for investments in capital, labour and the intermediate good are

$$r_t^K = \alpha(1-\zeta)\frac{Y_t}{K_t}, \tag{5.2.12}$$

$$w_t = (1-\alpha)(1-\zeta)\frac{Y_t}{H_t}, \tag{5.2.13}$$

and

$$Q_t = \zeta\frac{Y_t}{M_t}. \tag{5.2.14}$$

5.2.3 Households

Households like to consume and dislike work as per the utility function

$$U(C_t, L_t) = \frac{C_t^{1-\sigma_H}}{1-\sigma_H} - \chi \frac{L_t^{1+\eta}}{1+\eta}. \tag{5.2.15}$$

In addition, households are assumed to accumulate capital K_t and invest in deposits D_t. They decide consumption C_t, labour supply L_t, deposits D_t and new capital K_t by maximising their expected discounted lifetime utility

$$\max_{C_u, L_u, D_u, K_u} \mathbf{E}_t \sum_{u=0}^{\infty} \beta^u U(C_{t+u}, L_{t+u}) \tag{5.2.16}$$

where $0 < \beta < 1$ denotes the preferences discount factor. The household optimisation is subject to the budget constraint

$$C_t + D_t + K_t = w_t L_t + R_{t-1}^D D_{t-1} + (1 - \delta + r_t^K) K_{t-1} + \Pi_t \tag{5.2.17}$$

where δ is the depreciation rate of capital, r_t^K is the rental rate of capital, R_t^D is the gross deposit rate and Π_t is the profit distributed to households. Π_t is the combination of the profit distributed by the banks Π_t^B and the profit distributed by the intermediate good producing firms Π_t^M (the final good producing firms makes no profit and no loss)

$$\Pi_t = \Pi_t^B + \Pi_t^M. \tag{5.2.18}$$

Finally, one can derive the Euler equations for deposits and capital and the labour supply condition as follows

$$C_t^{-\sigma_H} = \beta R_t^D \mathbf{E}_t C_{t+1}^{-\sigma_H}, \tag{5.2.19}$$

$$C_t^{-\sigma_H} = \beta \mathbf{E}_t C_{t+1}^{-\sigma_H} (1 - \delta + r_{t+1}^K), \tag{5.2.20}$$

$$\chi L_t^{\eta} = w_t C_t^{-\sigma_H}. \tag{5.2.21}$$

5.2.4 Banks

Banks hold a balance sheet composed of loans issued to finance the intermediate producers' operations and fund these loans using households' deposits. The representative bank invests in a large enough portfolio of loans such as the final fraction defaulting is $\Phi(-\xi_{t+1})$, where $t+1$ is the loans' maturity and ξ_{t+1} the distance to default.[2] The bank recovers a fraction $1-\theta$ of the production proceeds after the borrower's default. In the case of the model I study, one can calculate the recovery term in the bank's profit function as follows

$$Rec_t := (1-\theta)Q_t g(X_{t-1}, X_{t-2}) \int_{-\infty}^{\infty} Z_{i,t}\mathbf{1}_{\pi^M_{i,t}<0} d\epsilon_i. \tag{5.2.22}$$

Given the defaulting fraction of loans $\Phi(-\xi_{t+1})$, the bank's expected profit from loans operations is

$$\pi^B_t = \mathbf{E}_t \left[\Phi(\xi_{t+1}) R_t X_t + Rec_{t+1} - R^D_t X_t \right]. \tag{5.2.23}$$

The profit function includes the return from the non-defaulting loans $\Phi(\xi_{t+1})R_t X_t$, the recovery from the defaulting loans Rec_{t+1} and the cost of borrowing from households $R^D_t X_t$. Banks are assumed to be competitive and face no entry cost so that the representative bank runs no profit and no loss in expectation. The zero expected profit condition yields a crucial link between the rates charged by banks and the expected default rates

$$R_t \mathbf{E}_t[\Phi(\xi_{t+1})] = \mathbf{E}_t \left[R^D_t - \frac{Rec_{t+1}}{X_t} \right]. \tag{5.2.24}$$

Clearly, the charged interest rate R_t increases with the default probability $\Phi(-\xi_{t+1})$, with the fractions of production lost conditional on default θ and with the deposit rates R^D_t. It is useful to note that, in the absence of defaults, the bank would charge the deposit rate when issuing loans to the intermediate production firms ($R_t = R^D_t$). The difference between the gross loan rate R_t and the deposit rate R^D_t reflects the extra spread banks charge the borrowing firms to compensate for losses due to future defaults. Credit spreads are defined as an annualised

[2] This is a direct consequence of the law of large numbers.

measure of the gap between default-risky and default-free gross rates

$$CS_t = \left(\frac{R_t}{R_t^D}\right)^{1/dt} - 1, \tag{5.2.25}$$

where dt is the time length separating two consecutive time periods ($dt = 0.25$ in the case of a quarterly frequency). The idea that banks charge the borrowing firms the expected cost of future defaults is made clearer by replacing for the loan rate in the expression of intermediate production firms' profit 5.2.6 using condition 5.2.24 to derive an expression for the profit expected by the borrowing firm

$$\mathbf{E}_t \pi_{i,t+1}^{M,d} = \mathbf{E}_t Q_{t+1} Z_{i,t+1} g(X_t, X_{t-1}) - R_t^D X_t - \theta \mathbf{E}_t Q_{t+1} Z_{i,t+1} g(X_t, X_{t-1}) \mathbf{1}_{\epsilon_i < \xi_{t+1}}. \tag{5.2.26}$$

The expression above shows that when there is no loss of intermediate production because of defaults $\theta = 0$, the intermediate producers' expected profits are uninfluenced by credit spreads or the probabilities of future defaults. The loan demand function remains impacted by credit spreads as the borrowing firms are price takers and the banks' pricing equation 5.2.24 is external to their profit maximisation problem. However, section 5.3 will show that, for the calibrated model parameters, credit spreads have little impact on credit markets when $\theta = 0$.

5.2.5 Aggregation and Market Clearing

In this subsection, I clarify the market clearing conditions. These are

i The clearing of the final goods market.

$$Y_t = C_t + X_t + K_t - (1 - \delta) K_{t-1}. \tag{5.2.27}$$

ii The clearing of the labour market.

$$H_t = L_t. \tag{5.2.28}$$

iii The clearing of the bank loan market where supply of loans by banks meets the demand of the intermediate production firms.

iv The clearing of the intermediate good market, taking into account the effect of bankruptcies in reducing intermediate production

$$M_t = \mathbf{E}_{\epsilon_i}[y_{i,t}^M] - \theta \mathbf{E}_{\epsilon_i}[y_{i,t}^M \mathbf{1}_{\epsilon_i < -\xi_t}]. \tag{5.2.29}$$

v The clearing of the deposits market

$$D_t = X_t. \tag{5.2.30}$$

The remainder of this paper is dedicated to the calibration and simulation of the model presented in this section. Section 5.3 presents the calibration and simulation procedures; and comments on the dynamic effects of each of the model's main assumptions.

5.3 Model simulations and findings

5.3.1 Steady-state equilibrium and calibration

In order to study the model numerically, I borrow the capital production function in Aksoy and Basso (2014) to express the deterministic part of the intermediate production function

$$f(X_t) = \ln(1 + X_t). \tag{5.3.1}$$

The steady state of the model exists and is unique for the set of model parameters chosen in our calibration[3]. The steady state default rate is $\Phi(-\bar{\xi})$ with a steady state distance to default given by

$$\bar{\xi} = \frac{1}{\Sigma_M} \ln\left(\bar{Q}\frac{f(\bar{X})}{\bar{R}\bar{X}}\right). \tag{5.3.2}$$

The quantity $\Sigma_M := \sigma_M\sqrt{1-\rho_M^2}$ is crucial to default rates in the steady-state. Given the model's remaining parameters, the volatility Σ_M is calibrated to match the historical U.S.

[3]See appendix 5.B for more on the steady state determination.

private default rate at 3.2%.[4] This default rate figure is consistent with Bernanke, Gertler, and Gilchrist (1998) and Pesaran and Xu (2016). The dislike for work parameter χ is chosen to match a steady-state labour at $\bar{L} = 0.3$. The loss in production following default θ is chosen to match the historical U.S. recovery rates at 40%. The preferences discounting parameter β is chosen so that the model's steady-state deposit rate matches the average historical deposit rates in the U.S. following Christiano, Motto, and Rostagno (2010). The parameter describing the share of intermediate goods in the production function is set to $\zeta = 0.5$ following Basu (1995) and Jones (2011a).

The cost of readjusting intermediate production λ and the correlation parameter ρ_M are key to the reaction of output, default rates and credit spreads following shocks. These two parameters are jointly calibrated to match output and credit spreads volatilities in U.S. data.[5] The remaining model parameters are standard. They are either chosen to match U.S. data or borrowed from the literature. Table 5.1 provides a summary of the model parameters, and table 5.2 provides the values of key steady-state variables. Table 5.3 shows the second moments of the main variables. The model replicates well the data's second moments of log output, log investments, log TFP and log credit spreads. On the other hand, the model implied consumption volatility is lower than the value inferred from the data. As we will see below, the low consumption volatility is inherited from the baseline RBC framework upon which the model was built and is not the consequence of the main mechanism studied here.

5.3.2 Dynamic effects

In this subsection, I show the dynamic effects of the model's main assumptions by studying the impulse response functions following negative shocks to the aggregate total factor productivity Z_t^a. I start by showing the effect of defaults in accelerating the business cycle before studying the effects of costly adjustment of intermediate production and the impact of the correlation

[4] See Moody's (2012) for Moody's quarterly default rates affecting U.S. private firms for the period between Q4 2002 and Q4 2011. Bernanke, Gertler, and Gilchrist (1998) assume 3% default rate in the steady-state while Pesaran and Xu (2016) assume 3.4%.

[5] More specifically, the volatility of credit spreads of BAA rated entities in the U.S., for the period between Q1 1980 and Q4 2011. Output and other macroeconomic data are collected for the same period. All macroeconomic time series are de-trended using a Hodrick–Prescott filter with a 1600 multiplier.

	Value	Source
Households preferences		
σ_H risk aversion	1	Christiano, Motto, and Rostagno (2010)
η curvature on labour	1	Christiano, Motto, and Rostagno (2010)
β discount factor	0.996	Christiano, Motto, and Rostagno (2010)
χ disutility of labour	9.55	steady state labour at 0.3
Technology		
α	0.36	Christiano, Motto, and Rostagno (2010)
ζ	0.5	Basu (1995) and Jones (2011a)
ρ_a	0.79	U.S. data
σ_a	1.1%	U.S. data
δ depreciation rate of capital	2.5%	Christiano, Motto, and Rostagno (2010)
Intermediate production		
Σ_M idiosyncratic volatility	2.1%	Steady state annual default rate 3.2%
θ loss of production upon default	60.1%	Steady state loss upon default 60%
ρ_M correlation with systemic factor	2.14%	ρ_M and λ are set to target a volatility of log spreads
λ cost of varying investments	10.1%	at 28.4% and the volatility of log output at 1.37%

Table 5.1: Assumed and calibrated model parameters.

Variable	Steady state value
Debt level $\bar{X}$	0.112
Output $\bar{Y}$	0.237
Consumption $\bar{C}$	0.088
Capital $\bar{K}$	1.47
Labour $\bar{L}$	0.3
Intermediary good $\bar{M}$	0.106
Deposit rate $\bar{R}^D - 1$	0.40%
Default Rate $\bar{DP}$	0.80%
Distance to default $\bar{\xi}$	2.38
Credit Spread $\bar{CS}$	2.13%
Price of intermediate good $\bar{Q}$	1.12

Table 5.2: Steady state variables.

Variable	volatility (data)	volatility (model)
Output	1.37%	1.37%
Investment	4.68%	4.85%
Consumption	0.78%	0.30%
TFP	0.95%	1.1%
Credit spreads	28.42%	28.42%

Table 5.3: Second moments of log variables: model vs data.

between the efficiency of intermediate production and aggregate TFP. The displayed simulations are realised, for quarterly time periods, in Dynare, using third-order approximations.

The effect of default rates and credit spreads

Figure 5.1 shows the impulse response function (IRF) of the main variables of the model after one standard deviation unpredicted negative shock to logarithmic TFP ($\ln Z^a$). The IRFs are shown for: (***i***) the main model as calibrated in section 5.3.1 (***ii***) a version of the model assuming no loss in intermediate production upon default ($\theta = 0$, dashed lines) and (***iii***) a simple RBC model with no intermediate good production ($\zeta = 0$, dotted line). It is important to note that the model with no production losses due to defaults assumes the same cost of adjusting intermediate production λ and the same correlation among intermediate producers ρ_M as the main model while the remaining parameters are recalibrated as in section 5.3.1 except for the parameter θ that is set to zero. This guarantees that all the model parameters, except for θ, are the same as in the main model. The assumption $\theta = 0$ implies no immediate loss in intermediate production because of defaults. Besides, figure 5.1 shows that despite an increase in default probabilities following a negative TFP shock, there is little fluctuation in credit spreads when defaults imply no loss in production ($\theta = 0$). Steady-state results show that when $\theta = 0$, the steady-state credit spread remains positive but is very close to zero.[6] There is therefore little impact on credit markets and future intermediate production because of credit spreads when $\theta = 0$. The differences between the reaction of the model variables under the main calibration assumptions and when θ is set to zero can be associated with the effects of defaults and credit spreads. On the other hand, the RBC model results represent the main model's behaviour in the absence of intermediate production and credit markets.

The presence of defaults related production losses implies a larger drop in loan levels, investments, capital, labour and output. When defaults impact credit markets, default probabilities increase following a negative productivity shock. This increase results from the combined effect of lower intermediate production efficiency ($\rho_M > 0$) and lower intermediate good's prices. Both

[6]When $\theta = 0$, the recovery is very close to the face value of the loan but remain less than the face value.

these effects depress the revenues of intermediate producers causing higher default rates. This impact on default probabilities has two main consequences. First, higher default rates increase the proportion of intermediate production lost because of the bankruptcy workouts. The loss of intermediate production causes a larger immediate drop in intermediate production and, as a result, depresses output, labour, capital investments and consumption further. Second, credit spreads increase as banks adjust the interest rates they charge intermediate producers to compensate for future defaults and loan demand is lower as the borrowing firms consider the higher cost of financing on their future profits. Higher financing costs depress the size of loans issued further and, in turn, worsen the impact of lower TFP on future output relative to the model where defaults do not impact credit markets.

The RBC model produces the familiar dynamics with an immediate drop in output, investment and hours following a surprise negative shock to productivity. The models that assume the existence of an intermediate production sector relying on loan financing display a reverse hump-shaped reaction of intermediate production and output. This shape is due to the drop in the size of the loans issued, which lowers future intermediate production and negatively impacts final production in the second period after the shock. The reverse hump-shape reaction of intermediate production to adverse shocks implies a similarly shaped reaction of hours worked as labour is less productive when fewer intermediate goods are available in the economy. In the current set-up, I assume a single-period maturity of the loans. A version of the current model with longer loan maturities would propagate the effects of TFP fluctuations for multiple periods after the shock. Longer maturity debt instruments would, therefore, cause more persistence in the response of output and investments.

Similarly to Carlstrom and Fuerst (1997), a positive autocorrelation of outputs is generated by assuming the possibility of bankruptcy in parts of the economy's productive sector. However, there is a major difference between the mechanisms of both models. In Carlstrom and Fuerst (1997) investments cuts are delayed following adverse shocks as it takes time for the shock to affect the net-worth of borrowers, thereby affecting their ability to raise external financing. In the model I study, the worsening of aggregate productivity triggers a systemic increase in default probabilities. Banks reflect higher future default probabilities in the credit spreads. The higher credit spreads discourage intermediate production firms from borrowing and reduce the level of loans in the economy. Which, in turn, reduces future intermediate production and future output. The model I present produces countercyclical credit spreads and default rates that delay part of intermediate production and output reaction to shocks. The countercyclical behaviour of credit spreads is a documented feature of business cycles that is not generated by models of the type in Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1998). As explained in Gomes, Yaron, and Zhang (2003), in these agency cost models, the borrowers' net worth takes some time to deteriorate after a negative shock. This dampens internal funds' reaction and implies a lower reliance on external funds as the financing needs drop faster than internal funds. Monitoring costs decrease as a result, thus reducing default premiums after negative shocks. On the other hand, the slow reduction in net-worth also pushes the borrower to delay some of the investment cuts, generating a reverse hump-shaped reaction of investments and output. Popular agency cost models' ability to generate a persistent autocorrelation of output and investment is tied to them displaying procyclical default premiums. Procyclical credit spreads are inconsistent with empirical observations regarding credit spreads and the business cycle.

Effect of investment adjustment costs

Figure 5.2 shows the dynamic effect of costly adjustment of intermediate production. The figure shows the impulse response functions for a calibration that assumes that no costs are incurred by intermediate producers when changing production levels ($\lambda = 0$). This "no-adjustment cost" model assumes otherwise the same correlation parameter ρ_M as the main model while the parameters χ, Σ_M and θ are recalibrated as in subsection 5.3.1. The parameter λ has no steady-state implications, so the steady-state calibrated parameters χ, Σ_M and θ are the same in the main calibration and "no-adjustment cost" calibration.

The presence of intermediate production adjustment costs primarily affects the reaction of default rates, credit spreads, and loan principals to a deterioration in the aggregate productivity, with a minor impact on the remaining model variables. The impact on default rates immediately after the shock is almost identical in the main calibration's model and the "no-adjustment cost" model. However, because adjustment costs hinder the borrowers' ability to set the demand for loan lower, their presence prevents demand for loans to drop at the same rate as in the "no-adjustment cost" calibration. The drop in the loan principal is less severe as a result. The relatively higher loan sizes imply that defaults remain high in the following period. This persistence of default rates increases the immediate reaction of credit spreads.

The way the parameter λ impacts credit spreads more than consumption, real investments and output, justifies its use to help the model match the historical volatility of credit spreads as per the calibration process described in section 5.3.1. As mentioned above, the costs of changing the size of intermediate production are not crucial to the model's ability to generate countercyclical default rates and credit spreads. However, by slowing the borrowers' reaction to adverse shocks, these adjustment costs make the reaction of default rates more persistent and is key in reproducing credit spread volatilities that are consistent with data.

Effect of the covariance between intermediate and final production

Figure 5.3 shows the impact of the correlation between the productivity of the intermediate producers and aggregate productivity (ρ_M) on the response of the main variables of the model to a negative aggregate productivity shock. The "high correlation" model assumes that the correlation parameter is ten percentage point higher ($\rho_M = 12.1\%$) and maintains the same intermediate production adjustment cost parameter as in the main calibration. The remaining parameters are recalibrated as in section 5.3.1.

A high intermediate efficiency correlation parameter $\rho_M = 12.1\%$ means that the efficiency of intermediate production deteriorates more following negative shocks to TFP and implies lower intermediate production. The larger drop in intermediate production implies a larger drop in output, causing lower investments, consumption and hours. Furthermore, a higher correlation parameter ρ_M worsens the deterioration of intermediate producers' revenues following negative shocks and causes higher default rates. This, in turn, causes higher credit spreads following adverse shocks.

The higher correlation parameter ρ_M causes a more considerable drop in intermediate production. This is only partially compensated by the smaller drop in the price of the intermediate good. As a result, intermediate producers' revenues drop further when ρ_M is high, thus causing higher default rates and credit spreads following a negative aggregate shock. Higher values of the parameter ρ_M imply higher volatility of output, consumption, default rates and credit spreads. These dynamic effects of the parameter ρ_M justify the calibration choice made in subsection 5.3.1, where the pair (λ,ρ_M) is calibrated for the model to match the historical volatilities of output and credit spreads.

As suggested by the low value of the calibrated correlation $\rho_M = 2.14\%$ in the main calibration, this parameter is useful to replicate historical values of the second moments of some of the aggregate variables but is not crucial to the functioning of the model's main mechanism.[7] Figure 5.4 in the appendix shows that even when the efficiency of intermediate production is independent of aggregate TFP ($\rho_M = 0$), default rates and credit spreads remain countercyclical and accelerate the business cycle by affecting loan issuance and intermediate production, thus causing output, consumption and hours to drop further. The countercyclical behaviour of default rates (and by extension that of credit spreads) is chiefly a consequence of the procyclical behaviour of the borrowing firms' revenues. This procyclical behaviour of revenues is rooted in the fact that the borrowing firms' good is an input in the final production process. Demand for the intermediate good drops during slumps, which negatively affects the intermediate good's price and, by extension, reduces intermediate firms' revenues. The lower revenues of the borrowing firms increase default rates and credit spreads.

[7] Alternatively to matching the volatility of log output, one can calibrate for the value of ρ_M to match the historical volatility of investments or hours.

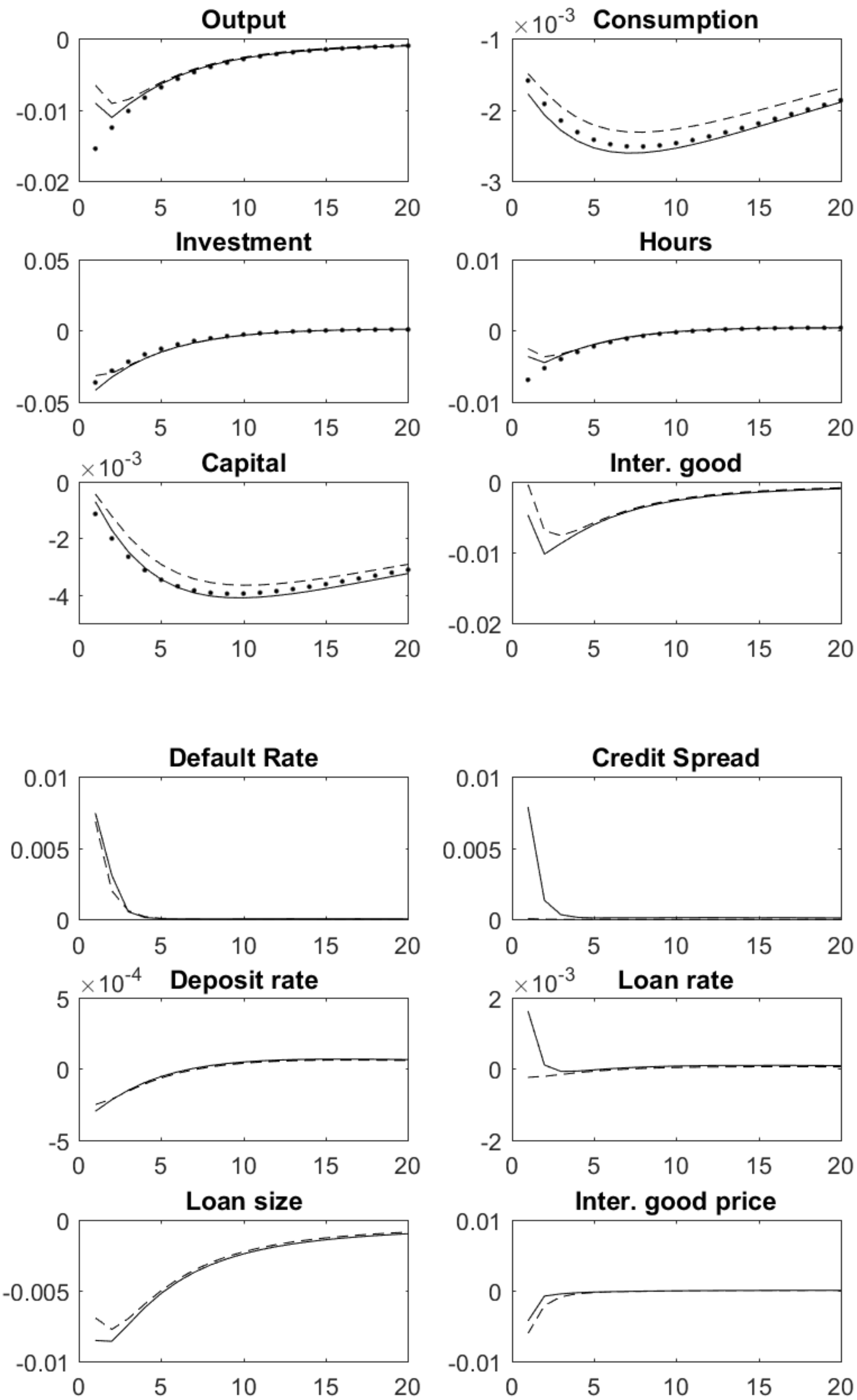

Figure 5.1: Impulse response functions following a negative shock to TFP ($-1\times$ standard deviation) of: (***i***) the main model, (***ii***) a version of the model assuming no loss in intermediate production upon default ($\theta = 0$, dashed lines) and (***iii***) a simple RBC model with no intermediate good production ($\zeta = 0$, dotted line). All variables but deposit rates, credit spreads and default probability are in logarithmic form. Credit spreads are annualised, but other interest rate variables are not.

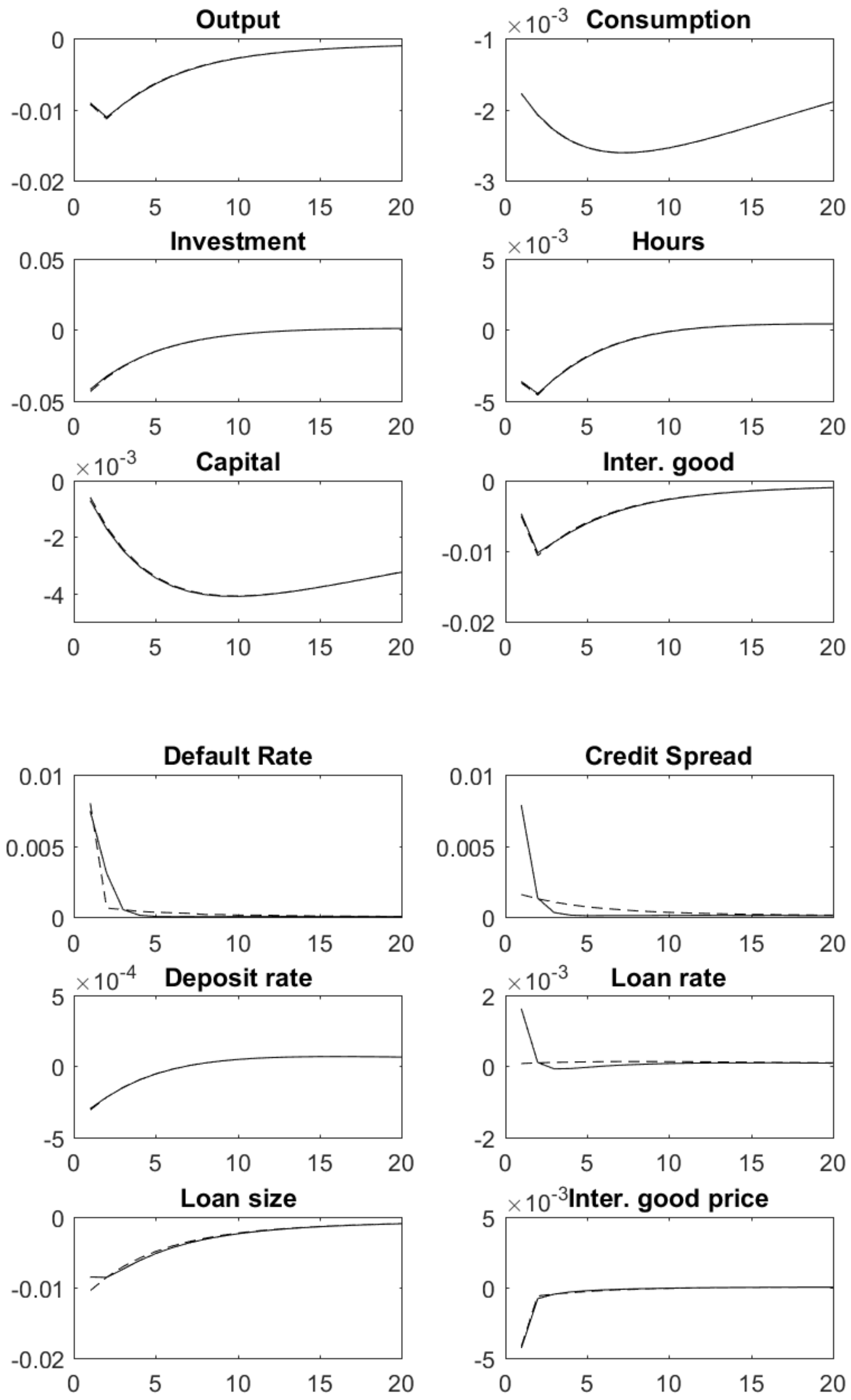

Figure 5.2: Impulse response functions following a negative shock to TFP ($-1\times$ standard deviation) of: (***i***) the main model and (***ii***) a version of the model assuming no cost of changing intermediate production ($\lambda = 0$, dashed lines). All variables but deposit rates, credit spreads and default probabilities are in logarithmic form. Credit spreads are annualised, but other interest rate variables are not.

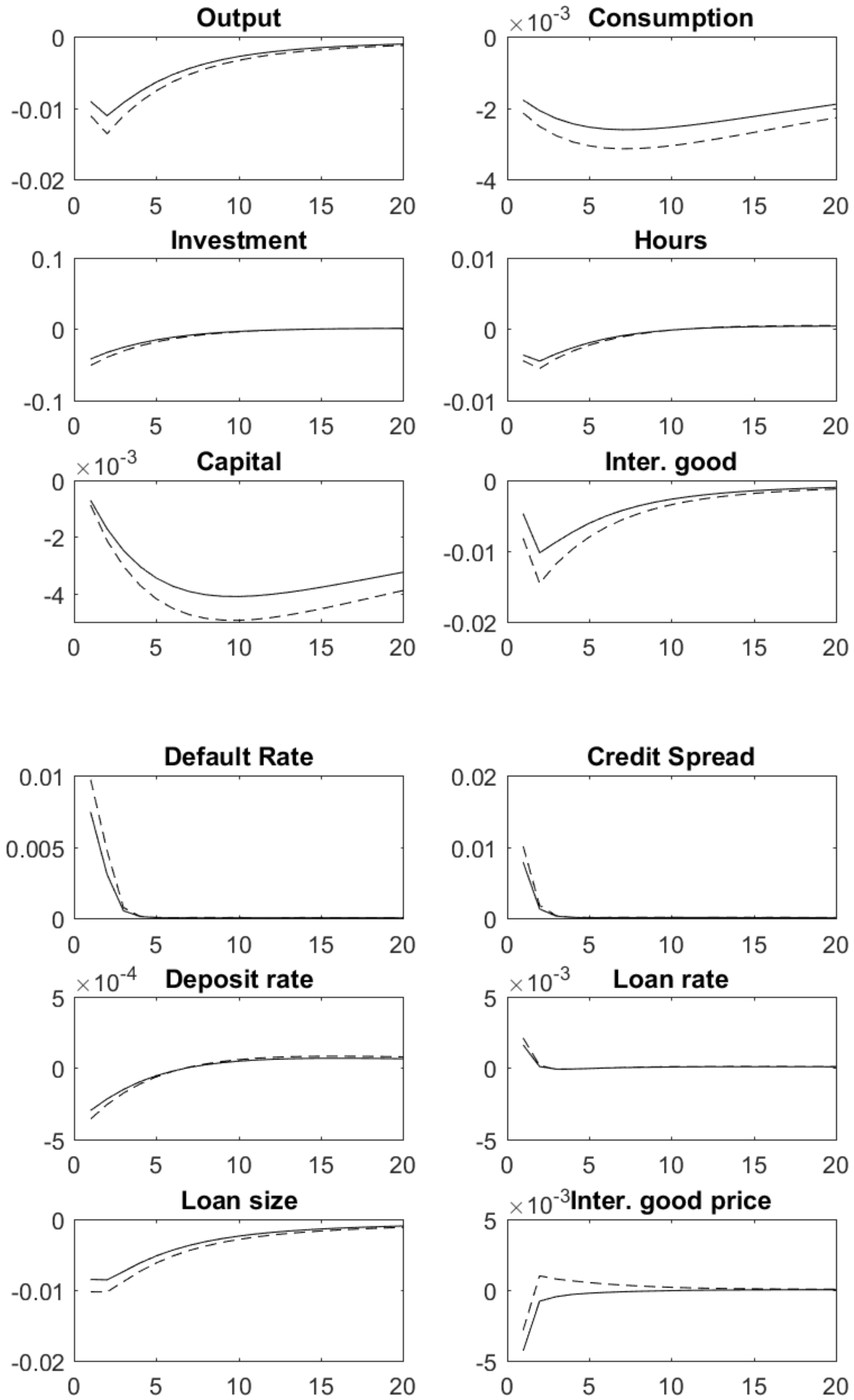

Figure 5.3: Impulse response functions following a negative shock to TFP ($-1\times$ standard deviation) of: (***i***) the main model and (***ii***) a version of the model assuming that the efficiency of intermediate production is more correlated with aggregate TFP ($\rho_M = 12.1\%$, dashed lines). All variables but deposit rates, credit spreads and default probabilities are in logarithmic form. Credit spreads are annualised, but other interest rate variables are not.

5.4 Concluding remarks

I present a general equilibrium model with endogenous defaults that reproduces the countercyclical fluctuations of default rates and credit spreads. This is achieved by assuming the existence of a sector of firms that borrow from banks to produce an intermediate good used by a representative final production firm. The model displays procyclical behaviour of the intermediate good prices. This price behaviour depresses the revenues of intermediate production firms during slumps and increases these revenues when the economy performs well, thus generating countercyclical default rates and, in turn, countercyclical credit spreads. The studied mechanism becomes more potent when assuming a positive correlation between the intermediate producers' efficiency and aggregate TFP. I also assume that the borrowing firms face a quadratic adjustment cost when adapting their production level to take productivity shocks into account. I show that these adjustment costs are key to generating quantitatively realistic dynamics of credit spreads, while the correlation between the productivity of the borrowing firms and the aggregate productivity contributes to the fluctuation of aggregate output, investment and consumption. These features inform the model's calibration process so that the model can generate reasonable dynamics for the aggregate quantities that are well captured by the usual real business cycle models while generating realistic dynamics of default rates and credit spreads.

The model considered in this paper provides a simple framework for the modelling of endogenous bankruptcies in a dynamic stochastic general equilibrium framework. The model can be extended to capture the behaviour of simple economies with long term debt contracts and economies with multiple debt maturities. Such modelling effort would help analyse the effect of long-term debt contract on the dynamic stability of the general equilibrium describing the economy and in the modelling of various other term structure effects.

Because of the relative simplicity of the used bankruptcy mechanism, it can be used in the context of larger general equilibrium models. For instance, monetary DSGE models can be augmented to reproduce countercyclical default rates and credit spreads and their effects on other aggregates by introducing an intermediate production sector relying on debt financing, as described in this paper.

5.A Model equations

The model equations describing the general equilibrium are presented in this appendix.

$$z_t^a = \rho_a z_{t-1}^a + e_t, \tag{5.A.1}$$

$$Z_t^a = e^{z_t^a}, \tag{5.A.2}$$

$$\xi_{t+1} = \frac{1}{\sqrt{1-\rho_M^2}} \left\{ \rho_M z_{t+1}^a + \frac{1}{\sigma_M} \ln \left(\frac{Q_{t+1} g(X_t, X_{t-1})}{R_t X_t} \right) \right\}, \tag{5.A.3}$$

$$DP_{t+1} = \Phi(-\xi_{t+1}), \tag{5.A.4}$$

$$\begin{aligned} R_t \mathbf{E}_t \left[\Phi(\xi_{t+1}) + X_t \frac{\partial \xi_{t+1}}{\partial X_t} \phi(\xi_{t+1}) \right] &= \frac{\partial g\left(X_t, X_{t-1}\right)}{\partial X_t} e^{\Sigma_M^2/2} \mathbf{E}_t \left[Q_{t+1} e^{\rho_M \sigma_M z_{t+1}^a} \Phi(\xi_{t+1} + \Sigma_M) \right] \\ &+ g\left(X_t, X_{t-1}\right) e^{\Sigma_M^2/2} \mathbf{E}_t \left[Q_{t+1} e^{\rho_M \sigma_M z_{t+1}^a} \frac{\partial \xi_{t+1}}{\partial X_t} \phi(\xi_{t+1} + \Sigma_M) \right]. \end{aligned} \tag{5.A.5}$$

$$Y_t = \left(Z_t^a H_t^{1-\alpha} K_t^{\alpha} \right)^{1-\zeta} M_t^{\zeta}, \tag{5.A.6}$$

$$r_t^K = (1-\zeta)\alpha \frac{Y_t}{K_t}, \tag{5.A.7}$$

$$w_t = (1-\zeta)(1-\alpha) \frac{Y_t}{H_t}, \tag{5.A.8}$$

$$Q_t = \zeta \frac{Y_t}{M_t}, \tag{5.A.9}$$

$$C_t^{-\sigma_H} = \beta R_t^D \mathbf{E}_t C_{t+1}^{-\sigma_H}, \tag{5.A.10}$$

$$\chi L_t^{\eta} = w_t C_t^{-\sigma_H}, \tag{5.A.11}$$

$$C_t^{-\sigma_H} = \beta \mathbf{E}_t C_{t+1}^{-\sigma_H} (1 - \delta + r_{t+1}^K), \tag{5.A.12}$$

$$Rec_t = (1-\theta) Q_t g(X_{t-1}, X_{t-2}) \int_{-\infty}^{\infty} Z_{i,t} \mathbf{1}_{\pi_{i,t}^M < 0} d\epsilon_i = (1-\theta) Q_t g(X_{t-1}, X_{t-2}) \Phi(-\xi_t - \Sigma_M) e^{\rho^M \sigma z_t^a + \Sigma_M^2/2}, \tag{5.A.13}$$

$$R_t \mathbf{E}_t [\Phi(\xi_{t+1})] = \mathbf{E}_t \left[R_t^D - \frac{Rec_{t+1}}{X_t} \right], \tag{5.A.14}$$

$$CS_t = \left(\frac{R_t}{R_t^D}\right)^{1/dt} - 1, \tag{5.A.15}$$

$$M_t = \mathbf{E}_{\epsilon_i}[y_{i,t}^M] - \theta \mathbf{E}_{\epsilon_i}[y_{i,t}^M \mathbf{1}_{\epsilon_i < -\xi_t}] = g(X_{t-1}, X_{t-2}) e^{\rho^M \sigma z_t^a + \Sigma_M^2/2}(1 - \theta\Phi(-\xi_t - \Sigma_M)), \tag{5.A.16}$$

$$D_t = X_t, \tag{5.A.17}$$

$$L_t = H_t, \tag{5.A.18}$$

$$Y_t = C_t + X_t + K_t - (1-\delta)K_{t-1}. \tag{5.A.19}$$

5.B Steady state

In this appendix, I provide the equations that uniquely determine the steady state of the model. First, the SS deposit and capital rental rates follow directly from the Euler equations

$$\bar{R}^D = 1/\beta, \tag{5.B.1}$$

$$\bar{r}^K = 1/\beta - 1 + \delta. \tag{5.B.2}$$

I will express the remaining SS variables as a direct or indirect function of the SS loan principal $\bar{X}$, the SS distance to default $\bar{\xi}$ and the SS price of the intermediate good $\bar{Q}$. First the SS intermediate good quantity is derived from the clearing condition 5.2.29

$$\bar{M} = f(\bar{X}) e^{\Sigma_M^2/2} \left(1 - \theta\Phi(-\bar{\xi} - \Sigma_M)\right), \tag{5.B.3}$$

where $\Sigma_M := \sigma_M \sqrt{1 - \rho_M^2}$. The intermediate good first order condition yields the SS output

$$\bar{Y} = \frac{1}{\zeta} \bar{M} \bar{Q}. \tag{5.B.4}$$

This and the first order condition for capital provides an expression for SS capital

$$\bar{K} = \frac{(1-\zeta)\alpha}{\bar{r}^K} \bar{Y}. \tag{5.B.5}$$

The final good clearing condition yields SS consumption

$$\bar{C} = \bar{Y} - \bar{M} - \delta\bar{K}. \tag{5.B.6}$$

Combining the first order conditions for labour provision 5.2.21 and demand 5.2.13 yields SS labour

$$\bar{L}^{1+\eta} = \frac{(1-\zeta)(1-\alpha)}{\chi}\bar{Y}\bar{C}. \tag{5.B.7}$$

One can then deduce the SS wages from the labour demand first order condition

$$\bar{w} = (1-\zeta)(1-\alpha)\frac{\bar{Y}}{\bar{L}}. \tag{5.B.8}$$

The recovery in the SS is

$$\bar{Rec} = (1-\theta)\bar{Q}\Phi(-\bar{\xi} - \Sigma_M)e^{\Sigma_M^2/2}f(\bar{X}). \tag{5.B.9}$$

The expression of the recovery combined with the bank loan pricing condition leads to an expression for the SS loan rate

$$\bar{R} = \frac{1}{\Phi(\bar{\xi})}\left(\frac{1}{\beta} + \bar{Rec}\right). \tag{5.B.10}$$

The SS credit spread are by definition

$$\bar{CS} = \left(\beta\bar{R}\right)^{1/dt} - 1. \tag{5.B.11}$$

The SS distance to default expression 5.2.4, demand for loans condition 5.2.8 and technology constraint 5.2.10 provide three equations to solve for $\bar{X}$, $\bar{Q}$ and $\bar{\xi}$ using the above to express other variables as a function of $\bar{X}$, $\bar{Q}$ and $\bar{\xi}$

$$\bar{\xi} = \frac{1}{\Sigma_M}\ln\left(\frac{\bar{Q}f(\bar{X})}{\bar{R}\bar{X}}\right), \tag{5.B.12}$$

$$\begin{aligned} \bar{R}\left[\Phi(\bar{\xi}) + \bar{X}\frac{\partial\bar{\xi}}{\partial\bar{X}}\phi(\bar{\xi})\right] &= f'(\bar{X})e^{\Sigma_M^2/2}\bar{Q}\Phi(\xi_{t+1} + \Sigma_M) \\ &+ f(\bar{X})e^{\Sigma_M^2/2}\bar{Q}\frac{\partial\bar{\xi}}{\partial\bar{X}}\phi(\bar{\xi} + \Sigma_M), \end{aligned} \tag{5.B.13}$$

$$\bar{Y} = \left(\bar{L}^{1-\alpha}\bar{K}^{\alpha}\right)^{1-\zeta}\bar{M}^{\zeta}. \tag{5.B.14}$$

5.C Impulse response function with $\rho_M = 0$

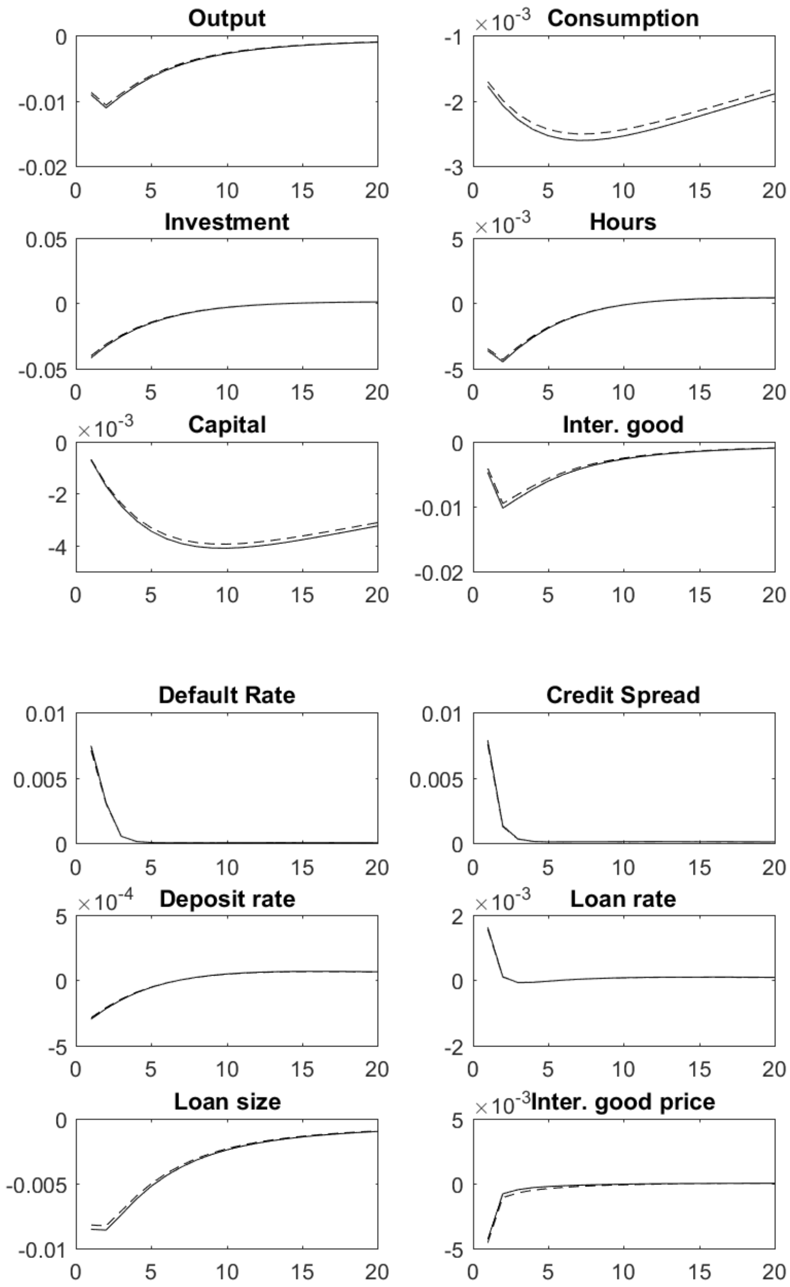

Figure 5.4: Impulse response functions following a negative shock to TFP ($-1\times$ standard deviation) of: (***i***) the main model and (***ii***) a version of the model assuming that the efficiency of intermediate production is independent of aggregate TFP ($\rho_M = 0$, dashed lines). All variables but deposit rates, credit spreads and default probabilities are in logarithmic form. Credit spreads are annualised, but other interest rate variables are not.

Chapter 6

Conclusion

Firms typically raise financing before fully realising their production costs, production efficiency and sale prices. A time friction arises because financing rarely happens under full information regarding the profitability of prospective projects. This thesis considers different aspects related to this time friction.

Chapter 2 considers the macroeconomic implications of firms spending less than the financial resources previously earmarked to fund the production process. The underspending mechanism is studied in a monetised RBC setup with firms raising financing before deciding their production spending. This study leads to several theoretical and numerical conclusions. The main theoretical conclusion is that the likelihood of firms entering an underspending mode depends on the size of the shock affecting the economy and the interest rate costs on existing debt. Numerical simulations are then realised, showing that the underspending mechanism asymmetrically affects the cycle as it only operates following large enough negative shocks. The simulation results also show that firms' underspending can significantly amplify the lows of the business cycle.

The underspending mechanism is studied in the context of a multi-industry model in chapter 3. This chapter proves that underspending can propagate from underperforming industries to the rest of the productive sector through input-output relations. The relationship between the likelihood of underspending and past interest rate levels is maintained in the multi-industry setup. In addition, I show that the nature of input-output relations plays a key role in maintaining the asymmetric effects of firms' underspending when the number of industries becomes large. Namely, industrial input-output networks where a few industries play an important role in supplying intermediate inputs to the economy are favourable to aggregations of industry-level shocks that keep the underspending mechanism operating.

Chapter 4 is an empirical study of the relationship between growth indicators and equity investors rewards. This chapter reinforces the existing literature concerned with the positive impact of lower growth opportunities on short term cash distributions to investors through dividends and share buybacks. For instance, I show that variations in firm-level TFP can contribute to explaining changes in dividends and share buybacks. Assuming that higher investor cash rewards imply low spending, this provides microeconomic empirical evidence that supports the underspending mechanism studied in chapters 2 and 3.

Chapter 5 of this dissertation considers a different mechanism originating at the same time friction causing firms' underspending. As firms raise financing before fully knowing all the factors affecting their future revenues, they can find themselves in situations where revenues do not cover existing debt obligations. This is used to build a general equilibrium model with endogenous bankruptcies and credit spreads. An intermediate producer/final producer structure is used to generate procyclical intermediate good prices and thus generating procyclical revenues for intermediate producers. The behaviour of revenues throughout the cycle means that the intermediate producers that borrow to finance the production of the intermediate input are more likely to default during the lows of the business cycle.

This work can be improved, extended and built upon in several ways. Below, I suggest few possible directions for extending and improving the content of this thesis.

- The underspending mechanism in chapter 2 can be provided with further empirical validation through the use of macroeconomic data sets. In addition, more theoretical research is required to disentangle the effects of the worsening of the investment opportunities from those related to precautionary saving and other competing effects.

- An obvious way to improve on the conclusions of the multiple-industry underspending model in chapter 3 is to fit it to real-life input-output networks connecting production in one of the advanced economies. This would allow studying the effect of the underspending mechanism on the aggregation of industry-level shocks in more realistic contexts than the ones considered in chapter 3.

- The endogenous defaults model in chapter 5 can be calibrated to match more aspects of default rates and credit spreads dynamics. For instance, one can attempt to match the volatility term structure of default rates. Moreover, the endogenous defaults model can be extended to include prices rigidities in line with modern DSGE literature. This would help design larger DSGE models that capture realistic dynamics of default rates and credit spreads without introducing an exogenous process driving credit risk.

- Combining the underspending effect in a multiple industry setup with chapter 5 endogenous defaults mechanism in a general equilibrium model can generate default dynamics with two distinct modes. When the size of adverse shocks surpasses the critical level beyond which firms start to underspend, default rates may jump much higher as the reduction in intermediate demand further depresses the revenues of the borrowing firms.

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